Effects of Noise in Time Dependent RWM Feedback Simulations

O. Katsuro-Hopkins, J. Bialek, G. Navratil

(Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY USA)

Building on the successful experiments on HBT-EP and DIII-D, active feedback control of the resistive wall mode (RWM) has emerged as an essential part of present and planned tokamak designs. In an effort to advance our feedback model closer to actual experimental conditions, the VALEN code was modified by introducing noise into the closed loop control system. In practice, feedback system performance is limited by detection thresholds and the stable operating range that is set by both systematic error and random noise in the measurement input to the feedback control loop. The results of an initial survey analyzing the effects of systematic error and noise (white, Gaussian, 1/f, etc.) on the RWM feedback system performance is performed using the newly developed time dependent capability in VALEN (see poster by J. Bialek).
Motivations

• Control of long-wavelength MHD instabilities using conducting walls and external magnetic perturbations is a very promising route to improved reliability and better performance of magnetic confinement fusion devices.

• It is well known that control of these resistive wall slowed kink modes above the no-wall beta limit is essential to achieve bootstrap current sustained steady-state operation in a high gain tokamak fusion energy systems.

• The ability to accurately model and predict the performance of active MHD control systems is critical to present and future advanced confinement scenarios and machine design studies. The VALEN modeling code has been designed and bench marked to predict the performance limits of MHD control systems.
• To enhance VALEN’s ability to model more realistic feedback systems initial value, time dependent capability was added to the code.
• To advance our feedback model closer to actual experimental conditions, the VALEN code was also modified by introducing noise into the closed loop control system.
• Understanding of non-ideal effects in feedback loop including systematic error and noise (white, Gaussian, 1/f, etc.) on the RWM feedback system performance is important for improved performance of tokamaks.
VALEN: A Reliable Computational Tool For RWM Passive and Active Control System Study

- **VALEN** is the only code available to model 3D external conducting structures with real porthole penetrations and other complex geometric features. Modeling of these features is crucial for accurately determining the passive stabilization properties of current and future devices.
- **VALEN** models unstable plasma modes using an electrical circuit representation developed by Boozer and plasma mode structure from DCON.
- **VALEN** models arbitrary external conducting structures using a 3D finite element electromagnetic formulation developed by Bialek.
- **VALEN** models arbitrary sensors, control coils and the control loop feedback logic that connects them to determine the effectiveness of feedback systems.
- **VALEN** predicts the growth rate of the instability, feedback currents and control system gain, and current distributions in the external passive structure to characterize RWM feedback performance.
- **VALEN** models initial value, time dependent problem.
- **VALEN** models non-ideal effects in feedback loop including systematic error and noise.
The VALEN Equations

The VALEN matrix equations describing the conducting structure and mode and control coil geometry are for the unknowns \( \{ I^w \} \), \( \{ I^d \} \), and \( \{ I^p \} \) are:

\[
\begin{align*}
[L_{ww}] \{ I^w \} + [M_{wp}] \{ I^d \} + [M_{wp}] \{ I^p \} &= \{ \Phi_w \} \\
[M_{pw}] \{ I^w \} + [L_p] \{ I^d \} + [L_p] \{ I^p \} &= \{ \Phi \} \\
[L_p] \{ I^p \} &= [S] \{ \Phi \}
\end{align*}
\]

The equivalent circuit (induction) equations describing the system mode growth are then:

\[
\{ \Phi_w \} + [R_{ww}] \{ I^w \} = \{ V \} \\
\{ \Phi \} + [R_d] \{ I^d \} = \{ 0 \}
\]

Where \( \{ V \} \) depends on sensor signals \( \{ \Phi_s \} \) via the feedback loop equations:

\[
\begin{bmatrix} M_{sw} & M_{sd} & M_{sp} \end{bmatrix} \begin{bmatrix} \{ I^w \} \\ \{ I^d \} \\ \{ I^p \} \end{bmatrix} = \{ \Phi_s \}
\]
• **VALEN uses DCON** (A. Glasser) results without a conducting wall to formulate the stability equation.

• Energy change $\delta W = 1/2 \sum \sigma_i \Phi_i^2$ in plasma & surroundings has negative eigenvalues $\sigma_i$ if an instability exists, $f_i(\theta, \varphi)$ diagonalizes $\delta W$ and defines the flux from the plasma instability $\Phi_i = \oint f_i(\theta, \varphi) \vec{B} \cdot d\vec{a}$.

• Complex helical magnetic geometry is expressed in terms of inductance and current $L_i = \Phi_i/I_i$ and the stability equation may be expressed as $S_{ij} = (\delta_{ij} + s_i \lambda_{ij})$ where $s_i = -\sigma_i L_i$ and the $\lambda_{ij}$ may be derived from the $f_i(\theta, \varphi)$.
VALEN Models External MHD Modes As Surface Currents

- The interaction of an external MHD plasma instability with surrounding conductors and coils is completely described by giving $\delta B_{\text{normal}}$ at the surface of the unperturbed plasma.
- VALEN uses this information in a circuit formulation of unstable plasma modes developed by Boozer to generate a finite element surface current representation of the unstable mode.

\[ \delta B_{\text{normal}} \text{ calculated by DCON for unstable plasma mode} \]

This methodology allows VALEN to use output plasma mode information from other instability physics codes (DCON, GATO, PEST or others).
VALEN's 3D Finite Element Capability Is Important In Accurately Modeling Passive Wall Stabilization Limits and Active Feedback Performance

- Correct representation of the geometric details of vacuum chambers with portholes and passive stabilizing plates is required to determine RWM control limits
- VALEN calculates these effects and allows the design of optimized control systems with complicated real-world machine geometry

Eddy current pattern induced in the control coils in the DIII-D tokamak

Eddy current pattern induced in the wall of the DIII-D tokamak due to an unstable $n=1$ RWM [top and side view]
DIII-D New Internal Control Coils are an Effective Tool for Pursuing Active and Passive Stabilizations of the RWM

- Inside vacuum vessel: faster time response for feedback control
- Closer to plasma: more efficient coupling
Internal Coils Provide RWM Stabilization by Feedback Control

- I-coil experiments have confirmed theoretical predictions that RWA is stabilized by direct feedback control
- Feedback performance is improved with internal poloidal field sensors
  - Faster time response
  - Decoupled from radial field of control coils
- Improved feedback performance is predicted for internal coils
  - Faster time response
  - Improved coupling to plasma
- Feedback stabilization up to ideal-wall limit requires that coil-wall coupling is not too large:
  \[
  C = \frac{M_{pw}M_{wc}}{L_u M_{pc}} \leq 1
  \]

In cylindrical model: for external coils \( C = 1 \)
for internal coils \( C = \left( \frac{r_c}{r_m} \right)^{2m} \)
Noise on the poloidal field sensors in the midplane. The signals are corrected for DC offsets. The power spectral density is shown as root-mean square amplitude per 10Hz frequency bin.
Simulated RWM Noise on DIII-D w/o ELMs

Low level noise was modeled as Gaussian random number with standard deviate 1.5 about 0 mean and frequency 10kHz
Simulated RWM Noise on DIII-D with ELMs

To the low level noise ELMs (Edge Localized Modes) were added as small group of Gaussian random numbers from 6 to 16 Gauss approximately every 0.01 sec with different signs +/- chosen with 50% probability.
Resonant Field Amplification

A. Boozer predicted Resistive Wall Modes would amplify magnetic field (e.g. errors see Physics of Plasmas 10, pg 1458 (2003)) The VALEN formulation handles this via an extra field source.

\[
\begin{align*}
\left[ L_{ww} \right] \{ I^w \} + \left[ M_{wp} \right] I^d + \left[ M_{wp} \right] I^p + \left[ M_{wD} \right] \{ I^D \} &= \{ \Phi^w \} \\
\left[ M_{pw} \right] \{ I^w \} + LI^d + LI^p + \left[ M_{pD} \right] \{ I^D \} &= \Phi \\
LI^p &= (1 + s)\Phi
\end{align*}
\]

Field amplification in the sensors is given by

\[
\begin{align*}
\left( \left[ M_{sw} \right] - \left[ M_{sp} \right] \frac{1 + s}{sL} \left[ M_{pw} \right] \right) \{ I^w \} + \left( \left[ M_{sp} \right] - \left[ M_{sp} \right] \frac{1 + s}{sL} L \right) I^d + \\
\left( \left[ M_{sD} \right] - \left[ M_{sp} \right] \frac{1 + s}{sL} \left[ M_{pD} \right] \right) \{ I^D \} &= \{ \Phi^s \}
\end{align*}
\]

We anticipate that noise signal applied to marginally unstable plasma will be amplified through the phenomena of Resonant Field Amplification.
Survey Feedback as Function of $\beta_{\text{normal}}$

Conversion from $s$ to $\beta$

$$\beta_{\text{normal}} = \frac{\beta - \beta_{\text{no\_wall}}}{\beta_{\text{ideal\_wall}} - \beta_{\text{no\_wall}}}$$

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DIII-D I-Coil Feedback model for the Control Coils L=60 $\mu$H and R=30 mOhm with Proportional Gain $G_p=7.2$ Volts/Gauss

Control coil current
Sensor Flux

Voltage applied to control coil
Maximum control coil current and voltage as function of $\beta_{\text{normal}}$
Proposed improved DIII-D I-Coil Feedback model for the Control Coils $L=10\ \mu\text{H} \text{ and } R=200\ \text{mOhm}$ with ELMs and with Proportional Gain $G_p=36 \ \text{Volts/Gauss}$

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**Control coil current**
Sensor Flux

Voltage applied to control coil
Maximum control coil current and voltage as function of $\beta_{\text{normal}}$
Effects of Noise on Feedback Dynamics for \( L = 60 \, \mu \text{H} \) and \( R = 30 \, \text{mOhm} \) DIII-D I-Coil Feedback model with Proportional Gain \( G_p = 7.2 \, \text{Volts/Gauss} \)
Control coil current

Voltage applied to control coil
Conclusions and Future Work

• Plasma resonant RWM response to noise increases closed loop current. Ideal limit can be approached but cannot be reached.
• Noise on the sensor flux does not effect stabilization dynamic of feedback.
• Voltage for the proposed improved DIII-D I-Coil feedback model is dominated by resistance of the connection cables, lowering resistance will significantly reduce the power requirements.
• Future Work:
  – investigate effect of noise on feedback dynamic with time delay
  – include noise in a full PID feedback control model
  – add current and voltage limits on feedback closed loop
  – add high and low frequency bandwidth limits to feedback closed loop.