Stochastic and physical modeling of motion of municipal solid waste (MSW) particles on a waste-to-energy (WTE) moving grate

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ABSTRACT
Numerical analysis of the mixing of municipal solid waste (MSW) particles as they travel on the grate of a mass-burn waste-to-energy (WTE) combustion chamber is necessary for understanding the parameters that control the combustion processes and designing the grate. In order to characterize the heterogeneous particle behavior, a 2-dimensional stochastic model of MSW particle mixing within a WTE combustion bed was developed. This model was calibrated and validated by means of a full-scale physical model of the Martin reverse acting grate, using tracer particles of sizes ranging from 6 to 22 cm. It was found that different particle sizes result in different residence times according to the Brazil Nut Effect (BNE). The motion of the reverse acting grate, in the speed range of 15–90 reciprocations/h, increases the mean residence times of small and medium particles by 69% and 8%, respectively and decreases that of large particles by 18%. Also, within this speed range, the mixing diffusion coefficient of each particle size was quantified. The ratio of particle diameter to the height of moving bar, d/h, was found to be a major parameter for the mixing diffusion coefficient and the particle residence time at reciprocations speeds exceeding 30 recip./h. Based on these quantitative results and the local MSW particle size distribution, the grate motion and the moving bar height can be designed for optimum operation.

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1. Introduction

Since municipal solid wastes (MSW) are extremely heterogeneous, the complex behavior of various MSW particles on the traveling grate of a mass-burn waste-to-energy (WTE) combustion chamber is one of the main concerns for controlling the combustion process. The movements of particles depend on MSW properties (such as particle size, shape, density, etc.), geometric parameters (such as design of grate system), and operational parameters (feed rate and reciprocation speed of moving bars). Therefore, modeling the behavior of each MSW particle is very complicated. Some important studies of mixing phenomena on a traveling grate have been conducted in the past, but they have not considered the importance of the size distribution of MSW particles.

The combustion chamber was considered as a combination of several continuously stirred reactors (CSRs) by Beckmann and Scholz [1]. In this bed model, an effective reaction coefficient was determined by data from a batch stoker test plant. Their research concluded that the model was successful for describing unsteady combustion of MSW. They also carried out residence time analysis using clay, wood and ceramic spheres in a small-scale model of the reverse acting and forward acting grates [2].

Three 1/15-scale models were developed for different types of grate system by Lim et al. [3] and the mixing process due to the motion of the grate was analyzed by following the movement of small cubic particles. One of the advantages of small–scale models is that they are easier to construct and operate. However, a disadvantage of using uniform size small particles rather than a distribution of actual MSW particle sizes is that it is not possible to accurately model particle interaction on the grate. Simplified mixing patterns in one-dimensional MSW bed layer model was applied by Ryu et al. [4]. In this model, mixing of the MSW bed was considered as an exchange between different layers in the bed on the grate. A model for the calculation of an unsteady, three-dimensional flow involving combustion phenomena in a packed bed of a solid waste combustion chamber was presented by Peters et al. [5,6]. This model simulated a bed comprised of MSW particles that mechanically interacted with neighboring particles. Based on the results of this study, they developed a model for mixing MSW...
particles on a forward acting grate using the Discrete Element Method (DEM). In the work of Yang et al. a diffusion model for MSW particle mixing was combined with a combustion model [7]. From their calculations, the mass loss rate at different mixing levels was investigated. The experimental results of their small-scale model and a full-scale real furnace test were compared and some discrepancies in diffusion coefficients were identified between the small-scale and full-scale tests.

Because of the size limitation, “as-collected” MSW cannot be used in small-scale models. On the other hand, full-scale grate models have not been used for examining solid waste mixing, because of difficulties in construction. However, a full-scale model would allow investigating the motion of actual “black bag” MSW collected from local communities. This is an important consideration since the motion of “as-collected” MSW samples on a full-scale grate can represent similar geometries and actual combustion chamber conditions of commercial WTE plants. For these reasons, in this study a full-scale model of a reverse acting grate was built and actual MSW particles were used in calibrated particle tracer tests. Also, for simulating the movement of each particle more precisely, a novel 2-dimensional stochastic model was applied to a mass-burn WTE grate system. It accurately modeled the experimentally observed enhanced particle mixing exhibited by actual MSW particles moving along a reverse acting grate.

2. Experimental setup

Particle tracers and an MSW bed were prepared. Spherical tracers of small (6 cm), medium (14 cm), and large (22 cm) sizes were made with gap filler insulating foam (Fig. 1a, left). These tracers were based on the particle size distribution of New York City (NYC) MSW particles: the mean value ($\mu$) of the size distribution was found to be 14 cm, where $\mu - \sigma \approx 5.8$ cm, and $\mu + \sigma \approx 22.8$ cm [8]. The density of the tracers was chosen to be 221 kg/m$^3$. This value was representative of the pre-compacted MSW and was lower than the reported typical value of about 297 kg/m$^3$ (500 lb/yd$^3$) [9] after compression. The MSW particles used in this study were obtained from black bags collected in New York City and were loaded to a height of 80 cm in the wooden apparatus shown in Fig. 1a, right. This apparatus is a full-scale physical section model of a reverse acting grate, having dimensions of 121 cm in length,

![Pictures of the sphere tracers with 3 different size and the apparatus](image)

![Geometry of the full-scale section model of reverse acting grate and MSW bed divided by 12 cells (20cm x 20cm)](image)

Fig. 1. Full-scale section model of a reverse acting grate.
91 cm in height, and 61 cm in width; the angle of inclination of the bed was 15°. Fig. 1b shows the geometry of the full-scale section model. The MSW bed was divided into 16 cells (4 layers x 4 sections), each of size of 20 x 20 cm. All four bars of the grate (2 fixed and 2 reciprocating) were 13 cm high and an angle of 14° to the grate inclination. The reciprocating bars traveled 42 cm from the top to the bottom positions, over a distance \(42 \cos 14° \approx 40.7\) cm that was approximately the same as the length of two cells (40 cm). Two digital cameras (model: Sony PCG-C1VR/BP and Sabrent SBT-WCCK), were positioned above and on the side of the apparatus and monitored the movement of tracer particles. This apparatus enabled the determination of the probabilities of particle movement in each vertical and horizontal position of the packed bed of actual NYC-MSW. After each reciprocation of the moving bars, the tracer positions were recorded by the cameras. Therefore, particles were traced from the original position to the position after \(n\) reciprocations. Because this physical model had neither an inlet nor a feeder, there was no MSW flow pushed into the chamber by a piston, as in actual operation of a grate. Therefore, in this “no-feeding” physical model, the motion of reciprocating bars and gravity were the sole driving forces causing the motion of the particles within the MSW packed bed.

Fig. 2. MSW bed on the reverse acting grate of a WTE plant showing divided cells (mesh) for the stochastic simulation.

Fig. 3. The elements of the flow matrix.
3. Mathematical model

The first procedural step in the design of the stochastic model was to divide the MSW bed into several cells (mesh grid shown in Fig. 2). In a commercial WTE plant, a reverse acting grate of total length of 6.4 m consists of 8 moving bars and 8 fixed bars and the MSW bed height is less than 1 m. In our mathematical model, the bed was divided into 32 sections along the axial direction and 5 layers along the horizontal direction, resulting in a total of 162 cells, including the inlet and ash bin cells. The particle movement probabilities were determined from experimental data and used to formulate a transition matrix $P$.

During the tracer tests using the physical model shown in Fig. 1, MSW particles are not changing in size (no burning) since this is a cold model. The stochastic process employed in the mathematical model is called a Markov Chain and it assumes that, at each reciprocation of the moving bars, the transition probabilities of the waste particles between adjacent cells are independent of the previous state in time. The rule governing the particle migration of the system is expressed by the following equation:

$$S(n) = (F^n \cdot P) \cdot S(0)$$  \hspace{1cm} (1)

Fig. 4. The corresponding cells and directions to the elements of the flow matrix $F$ and transition matrix $P$.

Fig. 5. The elements of the transition matrix.
where $S(n)$ represents the profile of MSW traveling on the chamber bed after $n$ times of reciprocations of the moving bars and $S(0)$ is the initial profile of MSW feed at the inlet (prior to any reciprocation of the moving bars, i.e., $n=0$). $F^k$ is the $k$th power of matrix $F$ that controls the MSW flow in the packed bed pushed by the feeder at the combustion chamber inlet. $k$ is the ratio between MSW feed flow rate and the frequency of the reciprocating bars. If there is no inlet feeder flow, then $k=0$ and $F$ becomes the identity matrix that neither affects $P$ or $S(0)$. The dimension of the flow matrix $F$ is 162-by-162. The elements of $F$ that we used are shown in Fig. 3 and their corresponding cells and directions are shown in Fig. 4. There are several different types of feeding systems, such as feeding bars and screws that are currently employed in mass-burn combustion chambers. In order to simplify the MSW flow caused by a feeder piston, we considered the motion of particles to be that of a plug flow. In this type of flow system, since particles travel only from section $i$ to the next section $i+1$ in a given layer, all transition probabilities $p_{i,i+1} = 1$ except for the exit location probabilities $p_{31,34}$, $p_{65,66}$, $p_{97,98}$, and $p_{129,130}$ (=0). This is because particles at the exit position in each layer (the 32nd section) cannot transition back to the inlet position for these 4 cells. All other $p_{ij}$ are equal to zero. Since an MSW flow greatly depends on the type of feeding system and chamber geometry, further study is needed to determine the elements of flow matrix $F$, specific to the particular operational conditions.

$P$ is the transition matrix that contains the probabilities predicting the solid particle movement due to the motion of the reciprocation bars. As shown in Fig. 5, the size of the transition matrix is 162-by-162 including a total of 26,244 probabilities, same size of flow matrix $F$. The main diagonal ($p_{1,1}$, $p_{2,2}$, ..., $p_{i,i}$, ..., $p_{162,162}$) elements represent the probabilities that the MSW particles remain in the same cell. As also shown in Fig. 4, $p_{i,i+1}$ is the probability that the particle transits from cell $i$ to the neighbor cell $i+1$ (along the flow direction) and $p_{i,i-1}$ is the probability from cell $i$ to cell $i-1$ (opposite to the flow direction). $P_{i,i+32}$ is the probability that the particle moves to the cell in the layer directly below the current cell (location from cell $i$ to cell $i+32$) and $P_{i,i-32}$ is the probability that it moves to the cell in the neighboring layer above (from cell $i$ to cell $i-32$). It should be noted that the probabilities in the transition matrix $P$ are always positive fractions and the sum of the elements within each row of the matrix $P$ equals one.

Since all of the solid waste enters at the inlet of the combustion chamber, the initial state vector $S(0)$ can be formulated as:

$$S(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

(2)

This initial state vector, whose size is $1 \times 162$, represents the state of the solid waste in the inlet cell, whose position is at the bottom of the hopper adjacent to the feeder bar.

The grate system of an actual combustion chamber has 8 moving bars as shown in Fig. 2, whereas our full-scale experimental cross-section model employs 2 moving bars. We determined the transition probabilities using measured data from the physical section model and we expanded the predictive capability of the stochastic model by employing 8 moving bars to match a full-length grate system. Based on the fact that the residence time of a real combustion chamber ranges from 30 to 120 min, a typical value for the MSW feed in this model was set at a time of

![Fig. 7. Visualization of particle path motion as a function of size: small particles (S), medium particles (M), and large (L) particles.](image-url)
64 min for traveling the total chamber length of 6.4 m (32 cells); i.e. 2 min/cell. In order to carry out the matrix calculations required in this stochastic simulation, MATLAB 7.1 was used on a Windows XP PC.

4. Results and discussion

Fig. 6 shows the measured probabilities of small, medium, and large particles that remain in the same cell location of an NYC-MSW packed bed after one reciprocation of the moving bars. The probabilities in layer E, at the bottom of the bed, are lowest for small and medium particles. For large particles, no experimental data could be measured in the bottom layer E because small and medium particles occupied it and tended to remain at the bottom so that large particles could not transit there. These probabilities mean that 22% of small particles (by particle number) and 8% of medium particles stay in layer E and the rest of them (78% of small and 92% of medium particles) move to a neighboring cell according to the motion of a reciprocating bar. Layer B is where small and medium particles have the second lowest probabilities and large tracers have the lowest. Approximately 50% for any one of the three sizes remain in layer B while the rest of them move to other cells. Retention probabilities for the middle of the MSW bed (layers C and D) are much higher with values up to 81%. Since the MSW is loaded at the height of 80 cm, layer B is at the top of the bed and a tracer on the free surface, which is the boundary between the solid (MSW bed) and gas (volatiles) phases, moves more because there are fewer particles acting on it to constrain its motion. Particles in this top layer were sometimes observed to be easily rolling along the free surface, a type of motion unavailable to particles in lower layers.

Numerical visualization of particle movement behavior that might be difficult or impossible to experimentally measure is particularly important in being able to understand and characterize a mixing phenomenon. Fig. 7 shows the calculation results of visualized particle movements for 12 particles traveling from section 1 through 32. Each particle travels along a different path in the MSW bed, but trends are consistent for a given size especially in the vertical (depth) direction. The general trend indicated by the simulation is that small particles tend to migrate downward (toward the grate) and large particles tend to migrate upward (toward the free surface) as they move along the bed. This size segregation is called the Brazil Nut Effect (BNE) and comes from the slight differences in vertical probabilities as a function of size. The segregation mechanism we observed in this study is illustrated in Fig. 8. The moving grates make space after one stroke from the top to the bottom position. Immediately, because of its unstable

Fig. 8. Brazil Nut Effect (BNE) in a packed MSW bed.

Fig. 9. C (top) and F diagrams (bottom) for small, medium, and large particles with a reciprocation speed of 90 recip./h.
condition, particles, whose size are less than the grate height, can easily drop into the empty space left by reciprocating bars as they move. Larger particles are less likely to drop into this newly created vacancy because their comparatively larger size offers more opposition to easily slipping into this limited empty space. After several reciprocations of moving bars, the size segregation appears as shown in Fig. 7. This mechanism of size segregation can be observed in rock, sand, powder, and granular movement. Some studies have been carried out employing this vertical selectivity mechanism as well as for electrical waste [10] as well as applying separation and recycle technologies to the processing of industrial waste and residential MSW.

**a small particles**

![C Diagram for small particles](image)

**b medium particles**

![C Diagram for medium particles](image)

**c large particles**

![C Diagram for large particles](image)

Fig. 10. C and F diagrams for (a) small, (b) medium, and (c) large particles: Dimensionless exit concentration C (left) and Dimensionless cumulative concentration F (right) versus residence time (min) with different reciprocation speeds ranging from 15 to 90 recip./h.
Fig. 9 shows the residence time distribution (RTD, dimensionless concentration versus residence time $t$) for small, medium, and large particles for a grate speed of 90 reciprocation/h. Residence time distributions of flowing materials were originally defined by Levenspiel [11]. The motion of the reverse acting grate increases the mean residence time of small and medium particles by 106 min (68%) and 69 min (9%), respectively, while decreasing that of large particles by 51 min (17%). In addition, two peaks of residence time distributions for small particles prominently appeared when the reciprocation speed exceeded 30 recip./h. As we discussed earlier, it is reasonable that the difference of mean residence times with size comes from the following reasons: (1) Small and medium particles are pushed by the reverse acting grate because their diameters are nearly the same or smaller than the height of the moving bars, $h$. (2) Larger particles are less likely to be caught by the grate so that their motion is less likely to be in opposition to the flow direction. (3) A mode of transport, rolling down the top surface along the flow direction, is available exclusively to those particles near the free surface of the bed. These two opposing motions at the bottom and the surface of the bed enhance MSW particle mixing and are responsible for the different residence time distributions. Due to the particle motion behavior (1)–(3), a vertical selectivity mechanism known as the BNE develops that tends to keep small particles in the bottom of the MSW bed. This effect is manifested by the presence of two peaks that appear only in the residence time exit concentration distributions of the small particles. The full data set of RTDs (C and F diagrams) for grate speed ranging from 15 to 90 recip./h is shown in Fig. 10.

Fig. 11 shows the mixing diffusion coefficient $D$ for different particle sizes and grate reciprocation speeds $R_r$. The mixing diffusion coefficient $D$ for all particle sizes increases linearly for speeds up to 30 recip./h. The $D$ for medium particles has a weak linear relationship with $R_r$, throughout the entire speed range from 0 to 90 recip./h. In contrast, the coefficient $D$ for large particles increases at a much slower rate and reaches a value of about 45 cm$^2$/min when the reciprocation speed reaches 90 recip./h. For small particles $D$ increases exponentially to 192 cm$^2$/min. When $R_r < 30$ recip./h, these relationships of grate reciprocation speeds $R_r$ and mixing diffusion coefficient $D$ can be described as follows:

$$D(R_r, d, h) = \begin{cases} \alpha \left(\frac{d}{h}\right)^2 R_r^{(1/3)} & \text{if } d < h \\ \alpha \left(\frac{d}{h}\right)^2 R_r^{(2)} & \text{if } d \geq h \end{cases}$$

(3)

Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m$^3$)</th>
<th>$D$ (cm$^2$/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaowood boards</td>
<td>225</td>
<td>109</td>
</tr>
<tr>
<td>Insulation bricks</td>
<td>480</td>
<td>88</td>
</tr>
<tr>
<td>Refractory bricks</td>
<td>770</td>
<td>27</td>
</tr>
</tbody>
</table>

where $\alpha$ is a constant ($\approx 1.1$) and can be considered to be a function of the number of moving bars. The constant $b$ has an approximate value of 2.8. The parameters $d$ and $h$ are the particle size and height of the moving bars, respectively, whose ratio $d/h$ controls the relationship between $D$ and $R_r$. The mixing diffusion coefficients range from 11 to 99 cm$^2$/min and monotonically increase for all particle sizes that varies reciprocation speed from 10 to 60 recip./h. These numbers represent the range at which the reverse acting grates are operated in commercial WTE Plants. This range of $D$ is in good agreement with the results of full-scale furnace tests in another study [7], shown in Table 1. In the full-scale test $D$ varies from 27 to 109 cm$^2$/min, though the type and operational conditions of the traveling grate are not specified. Though no mention was made of the tracer size, their densities were given as values 225 and 770 kg/m$^3$. Particle motion along the MSW bed is highly dependent on the experimental conditions such as the tracer size and density as well as physical properties of the bed itself.

As noted earlier, in real combustion chambers, the particle size is reduced because of volatilization and combustion processes, and the motion can be characterized as a reacting flow. Pushed by the reverse acting grate, small particles, which stay in the combustion chamber longer than medium and large particles, have a shorter time for complete combustion. According to a study of wood particle size versus conversion time, the 95% conversion time for small, medium and large sized particles at temperature of 900 °C is about 12 min, 46 min, 95 min, respectively (conversion time $\approx d^{0.62}$) [12]. The mixing diffusion coefficient $D$ for small particles is higher and they are easily dispersed along the horizontal direction. The advantage of this effect is that small particles that are mostly burned out can transfer their heat to large particles which need more heat and take more time to be thermally processed, especially during the drying and gasification processes. Primary air at room temperature comes from the bottom of the MSW packed bed. Due to the burn-out of the small particles pushed by the reciprocating bars and due to the high residence times at the bottom of the MSW bed, the primary air is warmed up by these small particles. The burn-out particles that turn into ash have a higher heat capacity than MSW and so they transfer heat to the large particles and the primary air. This study shows evidence that a grate speed does not affect burn-out of the entire MSW bed: The grate motion affects only small/medium particles whose diameters are less than the bar height, and whose mass volume is only 18% of total NYC-MSW [9].

5. Conclusions

A novel 2-dimensional model for the flow and mixing of the MSW fuel on the reverse acting grate was developed using a stochastic simulation. This model was used for the characterization and quantification of the mixing process. In this study, we found: (1) different particle sizes have different residence times according to the Brazil Nut Effect (BNE); (2) in the BNE larger particles rise to the surface while smaller particles migrate to lower depths of the bed where the reciprocating bars push backwards on the MSW with a net force along the bed opposing the direction of flow; (3) the motion of the reverse acting grate, in the speed range
of 15–90 recip./h, resulted in an increase of the mean residence time of small and medium sized particles by 68% and 9%, respectively, and a decrease of that of the large particles by 17%. (4) The bar height $h$ was found to be one of the major geometric parameters for the mixing diffusion coefficient $D$ and residence time when the reciprocation speed exceeds 30 recip./h.

This study provides a quantitative analysis to aid in the understanding of solid waste particle mixing during the combustion process in a WTE chamber. The results can assist in the evaluation of operational and geometric parameters of a reverse acting grate. The combination of stochastic and full-scale physical modeling can be a useful method for comparing and evaluating various types of traveling grate systems. This technique can also be a robust tool for designing a new generation combustion chamber. In order to improve this method, as future work, we suggest the following: (a) Examining other traveling grate systems, such as the forward acting grate and roller grate, by means of full-scale physical models and stochastic simulations and (b) analyzing mixing phenomena for different shapes, densities and sizes of MSW particles to more accurately describe the actual particle motion in the combustion chamber of an MSW packed bed.

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