Combustion and Emission Phenomena in Incinerators

Development of Physical and Mathematical Models of Incinerators, Part I: Statement of the Problem

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ABSTRACT

Combustion in an incinerator occurs in two differentiated zones: (1) the solid bed, Zone I, where the principal processes are pyrolysis, gasification, and combustion, and (2) the overbed region, Zone II, where the principal processes are flame holding and final burnup. Equations are developed to relate combustion intensity in each zone independently to the average combustion rate in each zone. This amplifies a previously developed equation relating the overall combustion intensity to the area firing rate and incinerator capacity. The basis for predicting limits to capacity and burning rate, by application of the thermal theory, is also outlined. This paper provides the basis for other papers in which additional approximate factors are developed for expanding the physical and mathematical model of an incinerator still further.

INTRODUCTION

A mathematical model of a physical system is a mathematical equation or set of equations, containing variables and constants, that can be used to predict the performance of the physical system. In electricity, for example, Ohm's Law is a simple mathematical model capable of predicting the current that will flow in a conductor of known resistance if a specified potential difference is applied. Similarly, the following incinerator design equation, is developed elsewhere [1]:

\[ F_A = \frac{c}{1/B} \times \left( \overline{T}/B \right)^{2/3} \times F^{1/3} \]  

(1)

where \( F_A \) is the area firing rate, \( F \) is the incinerator capacity, \( \overline{T} \) is the mean combustion intensity, \( B \) is the waste calorific value, and c is an incinerator (shape) constant. It is a very simple mathematical model of an incinerator. From this equation, as discussed elsewhere [2], \( F_A \) can be calculated for a selected value of \( F \) if \( \overline{T} \) is regarded as a constant for a given waste type, obtainable in principle from previously tabled values.

The range of conditions a mathematical model can predict depends on the assumptions and factors included in the model. Frequently it is found that so-called ‘constants’ in an equation are themselves variables depending on other, more fundamental parameters. For example, the electrical resistance required for calculating current flow using Ohm's Law depends on the conductor's dimensions, material, and temperature. If these factors are to be varied, the mathematical model must include supplementary equations to describe them. Similarly, in the incinerator design equation [Eq. (1)] factors omitted but capable in principal to inclusion include: bed depth, air rate, excess air, ratio of overbed to underbed air quantities, overbed air momentum, etc. These are all factors for which additional or supplementary equations can be written, as qualitatively outlined elsewhere [2], and all contribute to amplification of the model.
The detail a mathematical model can predict depends on how fundamental a level the analysis is taken to. At the broadest, most sweeping level, the analysis may be purely phenomenological — as in the case of the derivation of Eq. (1) — with relationships established only between gross input and output parameters. A more fundamental analysis must be based on mechanisms that are either assumed or proved to exist. Thus analyzing complex system generally leads to a set of differential and/or integral equations that have to be solved simultaneously. At that level of complexity, the solution almost invariably requires the use of computers, but if the analysis is very fundamental, a computer program designed, for example, to predict incinerator performance could in principle be constructed to describe local heat fluxes, temperatures, reaction rates, concentrations, smoke buildup, grit pickup, etc., as well as the gross parameter values such as chamber volume, shape, area burning rates, capacity, and combustion intensity.

The value of tested computer programs of mathematical models is that they can then be used to perform "computer experiments." In such experiments, the influence of some particular factor of interest can be examined, such as air rate, bed loading, or chamber shape (in the case of incinerators), if the program contains the desired parameter, by computing performance for a range of different settings of the selected factor. Ultimately, a computer program could be used to design incinerators for optimal performance. As a check on such a design program, it is always wise to test some of the predictions against selected experiments. However, the number of experiments then necessary can be substantially reduced — and if need be they could be dispensed with altogether — just as long as one is reasonably certain of the basic principles and equations and also of the couplings involved in putting the equations together.

This ultimate, of a design program, is a fairly long-term, but still potentially realistic, target. Computer programs of varying sophistication now exist for other combustion devices, notably jet engines and rockets, and there is no intrinsic reason why programs of equal sophistication should not be written for incinerators. However, a lack of certain information hinders writing such programs immediately, except for describing the gross overall behavior predicted by Eq. (1), and is a bar to greater sophistication. The accuracy of prediction by computations from a mathematical model is never any better than the accuracy of the physics and chemistry invoked in constructing the model in the first instance. This requires accurate information of two sorts, qualitative and quantitative. The qualitative information required is the nature of the mechanisms involved in the system (for example, whether the solid-bed reaction is dominated by chemical kinetics or physical diffusion), in addition to the equations governing the mechanisms. The quantitative information required is a table of reasonably accurate values of the coefficients involved in the equations, for without these the accuracy of prediction will diminish.

Always, however, we return to the prime proviso that models are never any better than the assumptions on which they are based; no model is reliable unless its component mechanisms have first been independently identified experimentally and unless the complete model incorporating the interacting mechanisms has then been adequately tested against suitable experiments. One set of experiments determines the nature of the mechanisms, and the other set substantiates the nature of the coupling between the mechanisms.

Determination of the component mechanisms has, therefore, been the subject of a research program at The Pennsylvania State University over the last 4 years. The target of this program as stated in the original proposal was: "To develop the information necessary to be able to design an incinerator for optimum performance from first principles. Optimum performance is specified as reliable operation with: (i) minimum particulate and objectionable gaseous emissions; (ii) minimum use of supplementary fuel; (iii) minimum construction cost; and (iv) minimum maintenance." In implementing this program, the first phase of the work has been directed, for reasons argued elsewhere [3], towards establishing the relationship between combustion behavior and combustible emissions. To determine individual mechanisms, a set of supporting subprograms was developed to study, independently, such factors as kinetics of burning waste materials, combustion in a solid bed, flame spread through a solid bed, combustion in a smoke flame, and overbed mixing behavior (by cold models). Using information drawn from these individual studies, the simple design, (or initial mathematical model) (Eq. (1)), has been amplified, and some of the quantitative amplification possible has already been described [2]. Finally, therefore, a check on the accuracy of the mathematical model being constructed is being performed by checking it against the performance of a simple test incinerator.

The position now reached on this phase of the
work is that the originally proposed target of a provisional mathematical model with an initial degree of experimental substantiation has been effectively achieved. The purpose of this and a set of associated papers [4-8] is to describe the model and its supporting experiments. Specifically, this initial paper sets the problem in context and identifies missing material. The four subsequent papers [4-7] describe the subprograms designed to fill the gaps. The final paper [8] uses this additional material to amplify the model still further and indicates the degree of substantiation so far reached in the experimental check using data from the test incinerator.

**QUANTITATIVE DEVELOPMENT FOR STABLE COMBUSTION**

In this section, our objective will be to amplify Eq. (1). As this equation stands, it can be used only one way (as described in more detail elsewhere [2]): if an incinerator is to be designed for a specific task, a known quantity of refuse must be destroyed per day, and this information is used to characterize the capacity, \( F \) (lb/h). Since the refuse type is known, then so also is its calorific value, \( B \) (Btu/lb). From current data on existing plants, typical values of \( T \) are known for various waste types. If an incinerator shape similar to any used previously is selected, the incinerator constant \( c \) is known (and is generally near unity in any case). This lists all parameters on the right side of Eq. (1); therefore, the only unknown, which is \( FA' \), can then be calculated. Since the grate area \( A \) equals \( F/FA' \), this then enables the grate to be sized. Since the combustor chamber volume is given by

\[
V_c = \frac{BF}{I}
\]  

this enables the combustor volume to be calculated. The effective chamber height (of length of the flame axis) can then be calculated from \( A \) and \( V_c \). Also, knowing \( F \), the total air rate can be calculated (with some appropriate value of excess air adopted).

Equation (1) can, therefore, be used for basic sizing of an incinerator, as long as the design adopted is not too different from any used previously, since \( I \) in this approach depends on past experience. However, the equation does not provide information on required bed depth, overfire to underfire air ratio, overfire air momentum, and other factors. For these quantities, supplementary equations can be written (otherwise one must depend as before on past experience). As we shall see, as the supple-

mentary equations are developed, we shall progressively break free of the straightjacket of dependence on past experience and, therefore, of dependence on existing designs as a basis for future developments.

**Zoned Combustion Intensities**

As a first step, we may regard any incinerator as consisting of two reaction zones. Zone I is the solid bed, and Zone II is the overbed region. For reasons discussed in some qualitative detail elsewhere [2], the solid bed is a zone where some combustion takes place, but most of the reaction is gasification with pyrolysis on the zone boundary. Only a fraction of the feed, at a rate \( F \) lb/h, actually burns to liberate heat in this zone, and the heat liberated must, of course, exceed the endothermic requirement for the gasification. Overall, this can be represented as some effective fraction, \( \theta \), of the feed burning with liberation of \( B \) Btu/lb. If the solid-bed volume is \( V_{I} \), then the average combustion intensity inside the solid bed \( I_{I} \), is

\[
I_{I} = \theta FB/V_{I}
\]

The balance of the waste must then burn in the gas phase in the overbed Zone II. If the volume of this section is \( V_{II} \), then the average overbed combustion intensity \( I_{II} \) is

\[
I_{II} = (1 - \theta)FB/V_{II}
\]

These two local average combustion intensities can now be related to the overall average intensity \( I \), given by Eq. (2). By eliminating \( \theta \) between Eq. (3) and (4) and then eliminating \( FB \), using Eq. (2), we obtain

\[
I = I_{I} (V_{I}/V_{c}) + I_{II} (V_{II}/V_{c})
\]

This is, in fact, a specific example of a general relationship that can be written

\[
I = \sum [I_{I} (\Delta V_{I}/V_{c})]
\]

where \( I_{I} \) is the local combustion intensity or heat release rate inside the local small volume \( \Delta V_{I} \); and this is also related to the local and overall (average) reaction rates since these are given by \( (I/B) \).

Equation (5) can be used to decouple the bed and overbed reactions. In principle, \( I_{I} \) and \( I_{II} \) can be calculated independently. If a bed depth is then chosen for some appropriate reason (to be discussed later), this determines \( V_{I} \) for given grate area. Similarly, \( V_{II} \) can be determined independently. This provides
the basis for calculating $\bar{I}$ instead of relying on past experience. It should also give us the parameters on which $\bar{I}$ chiefly depends so that we can determine whether or not $\bar{I}$ can be increased.

As a first approximation, however, it is of importance that $V_1$ is always relatively small compared with $V_{\text{res}}$, being generally 10 percent or less of $V_c$. We therefore have $(V_1/V_c) \simeq 1$, and from Eq. (4)

$$\bar{I} \simeq \frac{\bar{I}}{(1 - \theta)}$$

(7)

Since $\theta$ for gasifying refuse is likely to be as low as 0.1, the overbed combustion intensity alone is a good estimate of the overall combustion intensity. This, incidentally, was the assumption on which the analysis [1] leading to the estimate of the activation energy in the overbed reaction was based.

**Solid-Bed Combustion**

Analysis of the solid-bed behavior by itself provides expressions for two factors. One is $I_1$, for insertion in Eq. (5). The other is $F_A$ for substitution in Eq. (1). The equation for $F_A$ has already been derived previously [2] and has the form

$$F_A = \frac{(3/4)f_{\text{res}}(p_m \cdot \bar{G}_A)/(1 - V)}{(1 - (A + M))}$$

(8)

where $p_m$ is the oxygen mass fraction in air, $\bar{G}_A$ is the air supply rate, $V$ is the volatile-matter fraction of the dry, inert-free refuse, and $A$ and $M$ are the inert and moisture fractions, respectively. $f_{\text{res}}$ is a factor defined by Thring [9] known as the relative carbon saturation (RCS) factor. It represents the degree of potential saturation of oxygen by carbon, with a value of zero for pure air and a value of unity at the maximum saturation, which is carbon monoxide. This is a maximum mass saturation and should not be confused with a maximum valency saturation, which would be CO$_2$ for which $f_{\text{res}}$ = 0.5. The RCS factor can be calculated from the O$_2$ and CO$_2$ in the gas analysis at the top of the bed (if this can be determined without complications from the volatiles evolution) and is given by

$$f_{\text{res}} = \frac{[1 - 0.019(\text{CO}_2\% + 0.048(\text{O}_2\%)]/[1 + 0.010(\text{CO}_2\% + \text{O}_2\%)]}{(1 - (1/\text{res}) B)}$$

(9)

where $(\text{CO}_2\%)$ and $(\text{O}_2\%)$ are the volumetric or molar percentages of these two gases in the gasification products. This is the expression originally derived by Thring (see Ref. [10]) and constitutes a definition of $f_{\text{res}}$.

The RCS factor, if known, enables us to calculate the gasification rate, as Eq. (8) indicates (the method of determining $f_{\text{res}}$ is explained later in the paper), but does not give us the heat release in the bed since the factor is independent of any gas-phase reaction in the interstices between the particles in the bed. The extent of the gas-phase reaction may be determined from the complete gas analysis that includes CO. If a gas analysis ratio $r$ is constructed, defined by

$$r = \text{CO}_2\%/([\text{CO}_2\% + \text{CO}_2\%])$$

(10)

this ratio then represents the mass fraction of char residue in the refuse that burns only to CO, and $(1 - r)$ is the char fraction burning to CO$_2$. For 1 lb of char burning in this way, the total heat release will be $[(1 - B) C_0] + (1 - r) B_0]$, where $B_0$ is the heat combustion of the dry, inert-free char by itself and $n$ is the ratio of the heat of combustion of char burning to CO to the heat of reaction burning only to CO. Assuming the char to be primarily carbon, we have $n \simeq (94 \text{ kcal/mol})/(26.4 \text{ kcal/mol}) = 3.55$. Since the rate of char reaction is $[F_A(1 - V) - (1 - A - M)]$ (c.f., Eq. (8)), the heat release rate in the bed is

$$[F_A(1 - V) - (1 - A - M)] = (1 - V)$$

(11)

where the equivalence with the RHS is valid by virtue of the qualitative definition of $\theta$ given previously. Equation (11) therefore, constitutes a quantitative definition of $\theta$, which, by rearrangement, is given by

$$\theta = [(1 - r(1 - 1/n)](1 - V)$$

(12)

If the bed is so deep that the CO is very high and $r$ approaches unity, then $\theta$ is 0.1 or less when 100$V$ is 70 percent or higher, which is typical for cellulose-based materials such as paper.

These equations would, therefore, enable us to calculate $F_A$ and $\bar{I}$, if we knew or could calculate $f_{\text{res}}$ and $r$. For the RCS factor, an empirical expression relating this to bed depth already exists [9-11], as follows

$$f_{\text{res}} = K (1 - e^{-aL/d})$$

(13)

where $L$ is the bed depth, $d$ is an average particle diameter, and $K$ and $a$ are arbitrary constants whose values for a variety of different conditions are given in several references [9-11] (although there are some reservations to be made in applying the expression to refuse beds because of the substantially different nature and greater heterogeneity of the material).
This, therefore, defines one of the gaps to be filled, namely investigating $f_{es}$ as it varies with bed depth. The problem has now been solved for the uniform top-feed case by a general theoretical analysis deriving substantially Eq. (13). Similarly, the percentage ratio $r$ of Eq. (10) is obtained by the same analysis, for which there is not yet in existence any other empirical or other expression. The analysis leading to this result is given in one of the associated papers [4] of a set to which this paper belongs.

**Comparison of Zone Calculations**

It will now be evident that this approach through more fundamental considerations is substantially the inverse of that outlined previously for the direct use of Eq. (1). Instead of establishing first $I$, $F$, and $B$, to calculate $F_A$, this alternative approach is first directed towards the calculation of $F_A$ by Eq. (8). In the process, some appropriate bed depth $L$ is established, by use of Eq. (13) or its equivalent. If the bed is fixed with $F_A$ uniform across it, the grate area is determined as before in conjunction with specification of some capacity $F$. With bed depth and grate area established, this determines the solid-bed volume $V_I$ and all quantities in Eq. (3) can, therefore, be specified. There are still certain provisos and boundary conditions to be satisfied so there is not an absolutely free choice of bed depth, air rate, and other factors, but these are discussed in the consideration of boundary problems later in the paper.

The analysis of the overbed combustion, however, must be somewhat different. The problem here is determination of $V_{II}$ (or alternatively of the reaction zone or flame length $L_{II}$). Unlike the solid bed, where reaction is incomplete that there is some latitude in selecting $L$, $L_{II}$ is a quantity that is determined by the reaction rate of the overbed components. The problem can be specified in terms of the classic Rosin analysis (see e.g., Ref. [12]). If reaction rates can be calculated from appropriate theory, either absolute or operational, then the reaction burnout time can be calculated. In conjunction with the velocity profile or mean velocity, this can be used to calculate $L_{II}$ and, hence, $V_{II}$. The total combustion-chamber volume is then the sum of $V_I$ and $V_{II}$:

$$V_c = V_I + V_{II} \quad (14)$$

**Overbed Combustion**

The fundamental design and sizing problem in overbed combustion, as argued previously, is determination of reaction rates, since this information is needed to calculate $V_{II}$. Allied to this, however, are two other closely related problems of equal significance; these are the joint problems of overbed mixing between fuel and air and flame holding. These problems would also seem to be central to the practical problem of elimination of smoke. Inasmuch as the matter has already been discussed quite extensively elsewhere [1-3], a brief summary of the main points will suffice here. Smoke is the result of thermal cracking of carbonaceous fuels, generally in an oxygen-deficient atmosphere. Escape of smoke from a combustor can be the result of inadequate mixing with fresh oxygen after smoke formation and/or too low temperatures for smoke burnup after mixing. Alternatively, smoke will escape if smoke-forming volatiles leaving the top of the solid bed fail to ignite, i.e., if there is a problem of flame holding.

It was, therefore, argued [3] that the solution of the problems severally might be to utilize the Bragg's Criterion for ignition and burnout (in a jet engine [13]) and to form an aerodynamic flame holder above the solid bed by suitable design and location of the overfire air ports for injection of the secondary air. Since the objective of this approach is to generate the best practical approximation to a perfectly stirred section, it eliminates the inadequate mixing problem. If the stirred section also operates as a flame holder, as intended by design, this (within determinable limits) eliminates the problem of ignition failure. Finally, therefore, if the kinetic constants for the smoke residuals, volatiles, combustible gases, and others can be determined, then the burnup time can be calculated, and the overbed combustion volume, sized accordingly.

In passing, we may note that, theoretically, it is possible to specify circumstances such that the reaction rate would be so slow that the gases would cool to a point where reaction would effectively stop and again the unit could emit smoke. In practice, however, it is well known that with most waste types, the opposite situation is the problem. The gases are generally too hot, and for one reason or another excessively large quantities of excess air are added. If the excess air is reduced, there should generally be little difficulty in maintaining the temperature until combustion is complete, even if added fuel is necessary for very high-moisture wastes.

An analysis elsewhere [3], therefore isolated the problem of overbed mixing as central to the flame
holding and smoke burnup. To examine this problem, two subprograms were formulated, one using cold models to investigate the mixing process by itself without complications from reaction and the second including reaction by examining conditions of burnup of smoke in a smoke tube using one of the aerodynamic flame stabilizers (a countervortex system) examined in the model studies. The results of these two subprograms are given in Refs. [6] and [7].

This approach also specifies a general division, aerodynamically, into two gross regions (with fine detail ignored at this stage). These are idealized as a perfectly stirred region (Subzone II A) followed by a plug-flow region (Subzone II B). With this division, it is possible to apply Eq. (6) with $I$ defined separately for each subzone. By definition, for the perfectly stirred zone,

$$I_{IIA} = I_{IIA}$$ (15)

where $I_{IIA}$ is the local combustion intensity at any point inside the stirred volume. By definition of perfect stirring, this is the same at all points in the volume and therefore equal to the average.

For the plug-flow zone, the problem is somewhat different. Here the local combustion intensity is declining with distance (or time) through the zone as the fuel is burned up. Ideally, this can be calculated from first principles once the operational kinetic constants of the reaction are known. However, the combustion intensity will also be sensitive to heat losses in this region because of the relatively high temperature sensitivity of the reaction [1]. This is a problem that exists to some extent in the solid-bed zone, and also exists but is relatively easily handled in Subzone II A. As an interim measure, therefore, until the system is fully programmed, the intensity can be approximated empirically assuming an exponential decay in reaction rate. If a closer approximation is needed, a Rosin-Rammler exponential can be used. If we define $R_V$ as the local volumetric reaction rate and write this as $R_V = R_0 e^{-tr}$ where $R_0$ is the reaction rate at the margin between the two zones (at $t = 0$), with $r$ as an adjustable decay constant, then $R_0 = (I_{IIA}/B_0)$ since this is required for continuity. Integrating over the length of the plug-flow section to obtain $R_{II}$, we find

$$R_{II} = R_V B_0 = R_0 B_0 \left\{ 1 - \exp \left( -L_{II}/\nu \right) \right\} \nu \nu / L_{II}$$ (16)

where $\nu$ is an average velocity through the plug-flow region. If burnout is reasonably complete (sufficient to satisfy emission standards), the exponential term is negligible, and we obtain

$$\bar{I}_{II} = R_0 B_0 / (\nu \nu / L_{II}) = \bar{I}_{IIA} (\nu \nu / L_{II})$$ (17)

which is an interesting approximate simplification; the combustion intensity in this section falls inversely with reaction path length.

There remains then the problem of determining the relative volumes of the stirred and plug-flow sections. This is a complex problem, but it has now been solved in principle by a theoretical analysis developed elsewhere [14]. Application of the principle waits upon development of the kinetic constants for the smoke burnup.

**BOUNDARY PROBLEMS: IGNITION AND EXTINCTION**

Ignition and extinction are conditions of particular importance in incinerators since it is during these processes that units, particularly small batch units, are most likely to generate and emit smoke. This situation was determined qualitatively in a previous article [3]. In the discussion that follows the objective is to outline the basis of a quantitative analysis predicting onset of ignition or extinction. The relationship these conditions have to the steady-burning situation discussed previously is that of boundary conditions. Steady-state burning can be assumed as a basis for developing operating equations, but such an assumption contains nothing to indicate at what point the assumption becomes invalid and the operating equations no longer apply. Application fails on extinction; or, more precisely, the equations still apply but with unrealistically low combustion intensities and firing rates.

**General Nature of Problem**

The previous qualitative analysis of the problem [3] led to the identification of four reaction modes:

1. **Smoldering**, with slow reaction of the solid waste but no overbed burning. The rate of reaction is so slow that smoke production is often relatively slow, but the process is likely to be particularly odoriferous.

2. **Combustion**, with burning in the solid bed but not over the bed. This is the condition most productive of smoke.

3. **Burning (or flaming)**, when the bed is still hot enough to produce substantial pyrolysis products that burn over the bed. This is a less common
situation unless it is deliberately produced in a Dutch-oven type of combustor with no underfire air.

(4) Flaming Combustion, with ignition both in the bed and in the overbed zones. This is the most commonly desired situation if smoke is to be burned up.

In the four reaction modes, only in the last is there sustained ignition both in the solid bed and in the overbed zones. The other three reaction modes come about as a result of the combustion being extinguished in either one of the two zones or in both simultaneously. Fortunately, the analytical basis required to predict extinction is the same for both zones although the details of the application, naturally, will differ. This analytical basis will be summarized later, but first an additional distinction needs to be drawn between steady-state and unsteady-state operation.

Steady-state or equilibrium operation is identifiable as a situation where temperatures remain reasonably constant (maybe rising slowly or falling slowly) over a substantial period of time (say an hour or two). Such steady-state operation is then possible over some determinable range of input or ambient conditions, such as air rate, injection velocity, firing rate, moisture, etc. (with, naturally, different “steady”-state temperatures). If any one of the input or ambient conditions, e.g., air rate, is varied slowly, the system is constantly re-equilibrating, but, if the rate of change is slow enough, the deviation from true equilibrium can be so small that the system is always in “quasi-equilibrium,” and the steady-state equations still apply. A limit to the steady- or quasi-steady-state region always exists, however. If the reaction mode is, for example, flaming combustion, then at some very high or very low air rate (that is determinable in principle) the flame will extinguish. The extinction point itself is the limit to the steady state, but once extinction has occurred, all dependent variables such as temperature or gas composition change rapidly with time. This is the unsteady-state condition, which is also the condition when intense smoke emission is most likely.

Now consider the condition of initial light-off in a cold chamber. Initial ignition always requires an igniter, which can vary from a match applied to paper to a gas or oil torch. This can only be applied to a limited region of the bed, which adds another important dimension to the problem, that of flame spread (see Ref. [6]). With the flame spreading over and into the bed, the volatile products above the bed increase in concentration. At some point in time, their concentration will exceed a low limit, and self-sustaining combustion can be developed, as long as flame holding is not a problem. Before that moment, an overbed flame or igniter is required to burn up the initial pyrolysis products. This is rarely adequate, so initial light-off usually involves some escape of smoke. The most critical condition, however, is likely to be when the ignition torch is turned off (unless supplementary fuel is required throughout operation, for example, burning very high-moisture wastes). If overbed ignition then fails, smoke production can be copious.

In analyzing this particular situation, it is self-evident that we are, strictly, in an unsteady-state regime. In particular, the rate of fuel supply to the overbed flame increases fairly quickly as the reaction spreads through the solid bed, and the incinerator-wall temperature generally rises much faster than under quasi-equilibrium conditions. In spite of this, however, it is still possible, and to our advantage, to analyze the light-off by using steady-state approximations. In the first place, this eliminates the time-dependent components of all variables, which otherwise tend to increase complexity of analysis by an order-of-magnitude. In the second place, if analysis is based on a certain wall temperature and fuel-supply rate at a given moment in time, when in practice these parameters are actually a little higher, then a predicted extinction limit will give a conservative value, allowing some margin of error in actual practice. For this reason, in this and the set of associated papers, unsteady-state operation will not be considered in the analysis. Extinction conditions are examined in Ref. [4] for the solid-bed extinction and Ref. [6] for the overbed extinction. In addition, Ref. [5] contains an examination of the related topic of flame spread.

**Basis of Extinction Analysis**

The phenomenon of ignition and extinction in flame is one with a long analytical history reaching back at least a century, and one coherent theory of ignition that has now emerged from this is the thermal theory. According to this theory, ignition will occur in a critical finite volume element, and flame will then spread from that volume element, if two criteria are satisfied: (1) that the rates of heat generation by reaction and heat loss from the volume element are equal and (2) that the incremental rates of heat generation and loss with respect to temperature are equal. The origins and application of these criteria are summarized below.
This thermal theory of ignition – and extinction – is one with a substantial history of development and application, some of which has been reviewed fairly recently [15]. The basic idea of the theory is attributed [16] to Van't Hoff. It was formulated graphically by le Chatelier and mathematically by Semenov with significant extensions by Frank-Kamenetskii [16]; other extensions are listed in the review [15]. Most of these theoretical developments and applications refer to the unsteady-state situation with most ambient and dependent parameters changing with time. For our purposes here, however, the more pertinent analysis is that of the steady state, developed by Vulis [17]. It should be noted that the thermal theory is still not universally accepted, particularly for ignition of gases, because of the significant part played by generation of active species in initiation of chain reactions. Reconciliation of the thermal and chain theories of ignition is in progress [18], however, and in any case it would now seem that the generation and role of active species is most significant only in the case of auto ignition. The role of active species is either of lesser significance or unimportant in the case of reactor ignition, particularly ignition involving condensed phases.

The basis of the thermal theory of ignition and extinction can best be understood by considering Fig. 1. Suppose we have a finite element of volume either in the solid-bed or in the stirred region of the overbed space, and inside this volume element there is a steady throughput of fuel and oxidant. Since the fuel and oxidant react to some extent inside the volume element, some heat is produced, and, in Fig. 1,

![Diagram Illustrating Heat Generation and Heat Loss Curves as Basis for Classical Thermal Theory of Ignition and Extinction](image_url)

**Fig. 1** Diagram Illustrating Heat Generation $Q_I$ and Heat Loss $Q_n$ Curves as Basis for Classical Thermal Theory of Ignition and Extinction (Ignition and extinction temperatures are defined by tangency of generation and loss curves. $I$ = ignition point, $E$ = extinction point, $N$ = neutral or metastable point.)
The S-shaped curve \((Q)\) is a typical qualitative representation of how the rate of heat production varies with temperature. The precise, quantitative shape, of course, varies with different reactivity materials, concentrations, rate of throughput, and other factors, and it is thus that allows the solid-bed and overbed reactions to extinguish at different temperatures. In particular, the rate of heat production at the lower temperatures is generally so small as to be effectively zero.

Now consider the heat loss from the volume element. In a flowing system, gases enter the volume at one temperature and leave at a higher one, so that a convective heat loss occurs. In addition, there are also more direct heat losses by conduction and/or radiation to the surroundings, although these can often be reduced to minor proportions by adequate insulation. Typically, it is found that the rate of heat loss rises roughly linearly as the temperature of the volume element \(T\) rises above the surrounding temperature, \(T_s\). Figure 1 illustrates several possible heat loss lines \(Q_{\text{I}}\) to \(Q_{\text{II},4}\), for different flow rates (affecting the slope) and input temperatures (affecting the intercept on the \(T\)-axis).

Now suppose that the volume element is heated by any independent means to some arbitrary temperature \(T\). In general, it is found that the rate of generation \(Q_1\) differs from the rate of heat loss \(Q_{\text{II}}\). It is then obvious that, if the generation rate exceeds the loss rate, the temperature of the volume element will rise, but if loss exceeds generation, the temperature will fall. This moves the temperature of the volume element either to a high, ignited condition or to a low, unignited condition. The volume-element temperature finally stabilizes at the point where the \(Q_1\) and \(Q_{\text{II}}\) curves intersect. The stable operating temperature can, therefore, be predicted by writing down expressions for \(Q_1\) and \(Q_{\text{II}}\) in terms of the flow conditions and physical and chemical constants and then equating the two. Thus, for a first-order reaction of material entering a volume at concentration \(c_0\) and emerging concentration \(c\), with reaction in the volume governed by the Arrhenius expression: \(k_0 \exp \left(-E/RT\right)\), \(Q_1\) and \(Q_{\text{II}}\) can be written [17]

\[
Q_1 = k_0 c q \cdot \exp \left(-E/RT\right)
\]

\[
Q_{\text{II}} = (T - T_s) \bar{c}_p / r_s
\]

where \(q\) is the heat of combustion, \(\bar{c}_p\) is the mean specific heat of the products, and \(r_s\) is the residence time in the volume. The curves shown in Fig. 1 can be drawn by insertion of appropriate values in the constants. Equating and rearranging Eqs. (18) and (19) in dimensionless form, they become [17]

\[
\phi = \frac{1 - c/c_0}{c_s/c_0} = \frac{r_{sk}}{r_{sk}} \exp \left(-\frac{E}{RT}\right) = \frac{\theta - \theta_0}{\nu}
\]

where \(\phi\) is the completeness of reaction, \(r_{sk}\) is a dimensionless residence time \(= (k \tau_s)\), \(\theta\) is a dimensionless temperature \(= (RT/E)\), and \(\nu\) is a dimensionless input concentration \(= Rq_c/E\bar{c}_p\).

In determining the stable temperatures, it is evident from Fig. 1 that there will always be either one intersection or three. Where there are three, it can be shown [17] that the center intersection is a neutral or metastable condition. In most reacting systems, this is unimportant practically; it has primarily theoretical significance. Only one intersection means that the input and ambient conditions are such that either the system cannot be extinguished or else no ignition source of any size will ever light it.

**Definition of Ignition and Extinction**

The diagram is also used for defining ignition and extinction temperature. Consider, for example, the intersections of curve \(Q_{\text{I},3}\) with \(Q_1\) in Fig. 1. Suppose that the conditions are such that the system equilibrates at intersection \(A\) (unignited). There is potentially an ignited condition at intersection \(B\), but to reach that point the system must be moved from its stable condition at \(A\) round an "ignition barrier." This can be done in one of two ways. In the first place, it is obvious that the intersection \(A\) will move if the line \(Q_{\text{II},3}\) can be moved. Reference to Eq. (19) shows that this can be done either by increasing \(T_s\), the surrounding or input temperature, or by increasing the residence time \(r_s\). The second method moves \(Q_{\text{II},3}\) to a parallel position to the right. The second one rotates \(Q_{\text{II},3}\) to the right. Either way, a condition can be found theoretically (although not always practically) such that the \(Q_1\) line finally becomes tangent to the \(Q_{\text{I}}\). This is illustrated in Fig. 1 by the line \(Q_{\text{II},4}\). At the point of tangency, there is no longer a true intersection, and the temperature of the system is then able to rise through unsteady-state conditions to the upper stability point \(C\) on \(Q_{\text{II},4}\). The temperature corresponding to tangency of the two curves (at \(\frac{dQ_1}{dT} = \frac{dQ_{\text{II}}}{dT}\)) is, therefore, definable as the ignition temperature. If, finally, the initial input and ambient conditions are restored so that the heat loss corresponds to \(Q_{\text{II},3}\), the stability point moves from \(C\) to \(B\), and we have achieved our objective.

Extinction is similarly defined if conditions are readjusted further so that the heat-loss line moves or rotates further left to the point that a second tangency occurs, e.g., \(Q_{\text{II},2}\). The reaction then
extinguishes. Other possibilities for both ignition and extinction by adjusting other parameters are analyzed in some detail by Vulis [17].

The second method of ignition, to take the system again from A to B, introduces the concept of ignition energy. If the system is at A, then the whole system can in principle be raised in temperature by external heating (e.g., a torch) to a point just a little higher than \( N \) on line \( Q_{\text{g}} \). When the igniter is removed, the system temperature must continue to rise to \( B \) although the ambient and input conditions have remained unchanged. This can be regarded as jumping the system around the ignition barrier. It should also be noted that the system temperature (at \( T_{\text{N}} \)) must be higher than the tangency-defined ignition temperature \( T_{\text{i}} \) and the sensible heat input required to raise the system temperature to \( T_{\text{N}} \) is the ignition energy.

Experimentally, these two different approaches to ignition correspond to different physical operations. The conceptual process of moving the heat-loss line corresponds to adjusting the ambient or input conditions, by reducing the air supply rate to increase \( r_{\text{s}} \) or by preheating the air to increase \( T_{\text{o}} \). This latter, for example, can be done by burning gas or another fuel directly in the air being supplied under the grate. This, of course, reduces the oxygen concentration, but this may be more than offset by the increase in \( T_{\text{o}} \). In contrast, the conceptual process of jumping the system around the ignition barrier is achieved by direct heating of the reactants, for example, by the use of a torch on the bed itself. (In practice, heating of the underfire air also directly heats the bed itself). Which may be the faster method of ignition, however, is still unresolved. This is a problem not considered except in passing here, but if there is a significant difference in the time required for ignition, a comparative study of the two methods would be worth while inasmuch as this could reduce smoke emission during light-off.

It is also a significant point to note that extinction by massive heat loss can also occur by jumping round the extinction barrier as the parallel to jumping round the ignition barrier. This would correspond, for example, to dropping the system temperature from \( B \) to just below \( N \) on line \( Q_{\text{g}} \) and could occur physically if too large a quantity of very wet refuse were added to the burning bed at one time. This emphasizes again a point made earlier [2] about the need for devising means of providing uniform feed to incinerators, whether large or small.

These principles of ignition and extinction summarized in this section are developed further in the other papers [4-8] of a set to which this paper belongs by specific application to the two zones in the incinerator.

CONCLUSION

From examination of the analysis given, there are now two significant points emerging. The first is that proof of the validity of the equations developed could invert the whole approach to incinerator sizing in design since Eq. (8) enables \( F_{A} \) to be calculated directly, as a first factor, instead of using Eq. (1) for calculating it as a final factor. The second point is closely related. It is now clear from Eq. (8) that the waste-disposal rate of gasification rate is dominated or eventually determined by the solid-bed behavior; but the smoke-fume and odor-pollution problem is dominated by the overbed combustion since it is here that burnout is or is not finally completed. This is, of course, in agreement with intuitive concepts and with a qualitative assessment of the problem [2], but this is clarified by the semi-quantitative formulation.

This is explicit if Eq. (8) is written in the following form:

\[
F_{A} = K_{1} G_{A} \tag{21}
\]

where \( K_{1} \) is written for \([ (3/4) \rho_{\text{rcs}} \rho_{\text{pm}} ] (1 - V) [1 - (A + M)] \). \( K_{1} \) is not strictly a constant, but, for a given waste material and bed depth, it will approximate as a constant. Under such conditions, it is then seen that the rate of the waste disposal is directly proportional to the air rate. This remains true even in a Dutch-oven combustor with a solid floor; \( G_{A} \) then represents the average rate of air infiltration per unit area of floor covered. If, therefore, \( G_{A} \) must not exceed some predetermined value because of ash and grit pickup, say \( G_{A}^{m} \), then \( F_{A}^{m} = K_{1} G_{A}^{m} \) is the maximum area firing rate that can be allowed in the unit. There is, secondarily, still some latitude in adjusting \( F_{A} \) by changing the bed depth. If the bed is so shallow that the RCS factor is only 0.5 (generating only CO\(_{2}\)), then the substantial increase of bed depth to the point that the RCS factor approaches unity can almost double \( K_{1} \) and, therefore, double \( F_{A}^{m} \). In practice, however, most operational beds are likely to be of such depth that the RCS factor is unlikely to be much lower then 0.6, and increasing the bed depth is unlikely to increase it to much above 0.8. A more realistic variation in firing rate by adjusting bed depth is, therefore, more likely to be 30 to 40 percent. One other method of increasing capacity would be to accept incomplete burnout of the solid residue. An allowed combustible percent in the ash
could be quite high since combustion efficiency is not a factor of importance in waste disposal, and the residue would be sterile, so that it would not be a problem in sanitary landfill.

The overbed combustion problem is then the classic one of matching burning times to residence times, and, for the solution to that, kinetic data on the reaction of the smoke during burn-up must be developed.

In summary, therefore, the missing information required for more precise calculation of the volume requirements for burnup without smoke emission includes the following: (1) determination of the factors controlling the RCS \( f_{rc} \) factor and (2) determination of the kinetic constants for smoke burnup. Allied information required includes data on mixing effectiveness (stirring factors) and stirred-section volume to be sure of ignition and flame holding in the overbed region. Additionally, for light-off conditions, rates of flame spread and factors controlling this must be established so that the most effective light-off procedure can be developed that will bring a bed up to temperature fastest with a minimum of smoke. Finally, permissible limits for air rates and other factors must be established to be certain that there will be no loss of ignition in either the bed or the overbed zones, but more particularly in the overbed zone during initial light-off with cold walls. These are the factors being considered in lesser or greater detail in the associated papers of this set [4-8].

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