Modeling of Solid Waste Flow and Mixing on the Traveling Grate of a Waste-to-energy Combustion Chamber

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Abstract

Mixing of the highly non-homogeneous municipal solid wastes (MSW) on the traveling grate of mass-burn combustion chambers assists the combustion process in waste-to-energy (WTE) facilities. A matrix-based Markov chain model was developed to simulate particle flow and mixing as the solid waste particles travel over a reverse acting Martin grate. The model was used to project the pathway of a solid waste particle over a time series, in the bottom layer of the bed that is in contact with the bars of the grate. Further analytical and experimental work is planned in order to develop this model to a useful tool for designing future moving grate systems and increasing the combustion efficiency of existing WTEs.

1. Introduction

Simulation of the physical and chemical processes in moving beds is used widely to study combustion and other chemical reaction phenomena in gas-solid systems. Three general types of mathematical models are used for investigating the transport and chemical reaction phenomena in mass burn combustion chambers: Computational Fluid Dynamics (CFD), bed models, and stochastic models. CFD models simulate the fluid flow, heat and mass transfer, and reaction phenomena in the combustion chamber above the traveling grate by solving numerically the continuity and energy conservation equations and the Navier-Stokes equations (conservation of momentum). Numerical bed models of solid waste combustion have been developed since the early 1970s [1]. More recently, Yang, Swithinbank et al at Sheffield University [2] developed a two-dimensional program, called the Fluid Dynamic Incinerator Code (FLIC). FLIC is graphically interactive and is widely used by WTE engineers to simulate the physical and chemical transformations involved in the drying, volatilization and combustion processes on the grate; however, the combustion process is modeled using typical or average data such as composition, particle size, density and heating value, even though MSW is a very non-homogeneous fuel [3]. Stochastic models, such as the one described in this paper are relatively new, although some researchers have suggested combining mixing models of the traveling bed with experimental work [4].
Markov chain models have been used in the past to estimate mixing of powders in hoppers [5] and mixing in fluidized bed reactors. The present study is the first to apply the Markov chain model to the grate of a mass-burn chamber model in order to determine how solid particles move and mix on a moving grate.

A schematic diagram of the Martin reverse acting grate that was modeled in this study is shown in Figure 1. This grate is used widely and consists of one set of fixed and another of reciprocating grates. The motion and mixing of MSW particles over the grate are determined by various geometric and solids flow parameters as shown in Figure 1.

Figure 1: Reverse Acting Grate (Martin) [6, 7]: The reciprocating bars move against the direction of the feed by gravity: a) geometry of fixed and reciprocating bars, b) primary air (arrows) injected through the grate openings.

2. Modeling

2.1. Geometry of the reverse acting grate and solid waste flow

A typical Martin reverse acting grate is 7 meters long, consists of a total of 15 bars and is inclined 26 degrees to the horizontal. The bars are positioned at an angle of 13 degrees from the traveling bed. Eight of the 15 bars are reciprocating and move a distance of 0.42 m (420 mm). The frequency of motion of the reciprocating bars can be adjusted from 10 to 50 double strokes per hour. A typical operating frequency in a WTE unit is 12 strokes per hour but the frequency used depends greatly on the composition of the MSW feed.

A typical feed rate of MSW into the combustion chamber is 18.1 metric tons per hour (WTE of 480 short tons per day capacity). The average density of MSW is 500 lb per yd³ or 173.2 kg per m³ [8]. Therefore, the corresponding volumetric feed rate of MSW is 104.8 m³ per hour. A typical ratio of the
downward volumetric flow rate of MSW feed to the upwards flow of waste due to the motion of the reciprocating bars (volume of material pushed upward by the bar motion) is approximately 5:1.

2-2 Markov chain model

A model, based on the Markov chain model [9], was developed to simulate particle flow and degree of mixing as the MSW particles travel over the grate. The first procedural step in the design of the Markov chain model was to divide the solid waste on the bed into several cells ("state" of the system). Then, the transition of particles between these cells with time is examined. In the model, the entire grate was represented by 16 cells and by their respective boundary conditions (Figure 2). The computed data were compiled in the form of a transition matrix $P$ and a state vector $S(n)$ that describes the particle distribution over the states after $n$ transitions. The interchange of particles between successive cells ("Transition graph") is illustrated in Figure 3.

Figure 2: Illustration of the spatial disposition of cells on the reverse acting grate: Each cell represents either the reciprocating bar or the fixed bar.
Figure 3: Interchange of particles between successive cells (transition graph): The final cell of this model is cell 17 (ash bin); all particles in the MSW feed after combustion eventually arrive at cell 17, although the paths of individual particles differ considerably.

The Markov chain model assumes that, at each motion of the reciprocating bars (transition), the transition probabilities of the waste particles between adjacent cells do not depend on the previous state in time. The rule governing the particle migration (travel) of the system is expressed by the following equation:

$$S(n) = S(0) P^n$$  \hspace{1cm} (1)

where \( S(n) \) represents the profile of MSW traveling on the chamber bed after \( n \) times of double strokes of the reciprocating bars and \( S(0) \) the initial profile of MSW feed at the inlet (prior to any stroke of the reciprocating bars, i.e., \( n=0 \)); \( P^n \) is \( n \)-th power of \( P \) (called the \( n \)-step transition matrix) of the matrix that contains the probabilities of the solid particle movement. The latter is controlled by feed rate, particle size and density, geometry of the grate and frequency of reciprocating bars. The transition matrix \( P \) for the movement of the solid waste particles corresponding to the series of cells shown in Figure 3 is defined as follows:

$$
P = \begin{bmatrix}
\rho, & 1 - \rho, & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \rho, & 1 - \rho, & -\rho \\\n\end{bmatrix}
$$  \hspace{1cm} (2)
where $p$ is the probability that MSW particles will remain at the same grate position (cell) and $r$ the probability that the particles transit the next grate position (cell). The subscripts $f$ and $r$ represent fixed bars and reciprocating bars, respectively, the subscripts $i$ and $o$ represent inlet and outlet, respectively.

The movement of the reciprocating bars causes an upward motion of the solid waste particles against the downward direction of the feed. Thus, there is a transition from position/cell $k$ ($k$ is the cell number and is a positive integer, $1 < k < 16$) to the two adjacent compartments $k - 1$ and $k + 1$ (Figure 3). If $k$ is an even number (i.e., cells #2, 4, 6, 8, 10, ..., 14), cell $k$ represents a reciprocating bar. If $k$ is an odd number (i.e., cells #3, 5, 7, 9, ..., 15), cell $k$ represents a fixed bar. In order to simulate the mixing phenomena on the actual grate, six probabilities are defined at each cell:

- The probability $p_i$ of particle remaining in state 1 (cell #1).
- The probability $p_r$ of particle remaining on one of the reciprocating bars (cell #2, 4, 6, ..., 14).
- The probability $r_r$ of transiting to a reciprocating bar (cell #2, 4, 6, ..., 14).
- The probability $p_f$ of remaining on a fixed bar (cell #3, 5, ..., 15).
- The probability $r_f$ of transiting to a fixed bar (cell #3, 5, ..., 15).
- The probability $p_o$ of remaining in outlet state (cell #16).

It should be noted that the probabilities $p$ and $r$ are always positive fractions and the sum of the elements within each row in the matrix $P$ equals one. The values of $p$ and $r$ depend on the feed rate $R_f$, particle size of MSW components $S$, particle density of MSW components $\rho$, state of combustion, frequency of reciprocating bars $R_r$, the length of the reciprocating bars travel $L$, angle $\alpha$ of chamber bed decline, the height $H$ and the angle $\theta$ of the reciprocating bars. The relationships between probabilities and operating parameters can be expressed as follows:

- For the reciprocating bars (cells #2, 4, 6, ..., 14),
  \[ p_r \sim f_i(R_i, S, \rho, R_r, L, \alpha, H, \theta) \]  
  \[ r_r \sim g_i(R_i, S, \rho, R_r, L, \alpha, H, \theta) \]  
  \[ p_r + r_r < 1 \]  

- For the fixed bars (cells #3, 5, ..., 15),
  \[ p_f \sim f_f(R_f, S, \rho, R_r, L, \alpha, H, \theta) \]  
  \[ r_f \sim g_f(R_f, S, \rho, R_r, L, \alpha, H, \theta) \]  
  \[ p_f + r_f < 1 \]  

The relationships between probabilities are:

- For cell #1, solid waste particles are disposed to move only downward to cell #2, because cell #1 is next to the inlet and new feed enters all the time pushing the older particles downward the grate. Therefore,
  \[ p_i < 1 - p_i \]

- For cell #2 (also, cell #4, 6, ..., 14), although the reciprocating bar moves up and down, particles that ride this bar have less chance for transition from cell #3 to cell #2 and are disposed to remain in cell #3 (same as cells #5, 7, 9, ..., 15). Therefore,
  \[ r_r < 1 - p_r - r_r \]

- For cell #3 that represents a fixed bar (same as cells #5, 7, 9, ..., 15) particles have a high probability to move to the higher cell #2 (same as cells #4, 6, 8, ..., 14) because the motion of the reciprocating bar below cell #4 pushes particles upward. Therefore,
  \[ r_f \leq p_f < 1 - p_f - r_f \]  

The assumed probabilities for satisfying equations (9), (10) and (11) are shown in Table 1. At this stage of the study it was not possible to define the functions $f_i$, $f_f$, $g_i$ and $g_f$ in equations (3), (4), (6).
Table 1: Assumed values of probabilities satisfying equations (9), (10), (11) for the transition matrix used in this calculation

<table>
<thead>
<tr>
<th>Probability</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>0.4</td>
</tr>
<tr>
<td>$P_e$</td>
<td>0.5</td>
</tr>
<tr>
<td>$P_f$</td>
<td>0.3</td>
</tr>
<tr>
<td>$P_o$</td>
<td>0.3</td>
</tr>
<tr>
<td>$r_i$</td>
<td>0.2</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

and (7), because there has not been any experimental study of the mixing phenomena. Further work will be required to estimate values of these parameters, define Equations (3) to (8), and then validate the model projections by field tests in an industrial WTE. Since all the solid wastes enter at the inlet of the combustion chamber, the initial state vector $S(0)$ can be formulated as:

$$S(0) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (12)

This initial state vector represents the state of solid waste in cell #1, the position next to the inlet just after the solid waste fed by the pusher at the bottom of the hopper. As stated earlier, the present model examined only one layer of MSW, that in contact with the grate surfaces.

3. Results and Discussion

Figure 4 shows a simulated “random walk” that represents the particle movement based on a calculation over 46 steps (the required number of calculations varies between different time series because of the stochastic nature of the simulation results). Cell #1 represents the feed end of the grate and cell #16 the end of the grate where the ash falls to the ash pit below. Cell #17 denotes the state of bottom ash. This graph shows how a hypothetical solid particle travels over the reverse acting grate. At each motion of the reciprocating bar, the particle may remain in the same cell or move to the adjacent cell, downward or upward, due to the upward motion of the bars. At the #2 and #5 cells, the particle seems not to move for a while, which, in a several layer model of the bed on the grate, would result in mixing with the next layer up. The lower graph in Figure 4 provides a visual illustration of the behavior of a particle on the moving grate, based on the results of the matrix calculation. The particle actually moves back and forth between cells as indicated by the solid line. For example, between cells #11 and #12, the particle travels back and forth because of the action of the reciprocating bar.
Figure 4: Simulation results of the movement of a solid waste particle on the Reverse Acting Grate:  
Top graph: tracing a behavior of one particle on the bed. In cells #2 and 5 the particle seems to stay put for a while. In cells #11 and 12, the particle goes back and forth because of the movement of the reciprocating bar. Bottom graph: Visualization of the particle travel based on the results of the calculations shown on top graph.

Figure 5 shows the residency distributions of solid waste particles with successive motions of the reciprocating bars. Under the combination of downward gravity and upward motion of the reciprocating bars, all particles travel gradually to the outlet. The peak of each particle distribution profile is displaced toward the outlet and the profile becomes much flatter. The solid particles that move slower (particles in the left part of each profile in Figure 5) have been that have been pushed upward by the reciprocating bars. This action contributes to mixing partly combusted material with newly fed solid wastes.

It is evident that solid wastes that are more mixed have a better chance of being combusted completely. On the other hand, solid particles that travel faster than the bulk of the solid wastes (particles in the right part of the each profile in Figure 5) will not burn completely. Without additional feed coming in, the probability of particles staying in a specific cell, after an infinite number of movements (step \( n = \infty \)), approaches zero, because all solid particles travel toward the outlet and eventually reach it.
Figure 5. Change in tracer particles distribution profiles over grate with number of movements of reciprocating bars (n=5, 10, 40, 80). The regular perturbations in the n=40 and n=80 profiles are due to the effect of alternatively passing over stationary and reciprocating bars.

4. Conclusions

A new method for investigating mixing on the grate of a WTE combustion chamber, using the Markov chain model, has been presented. This method may become a good tool to predict the actual path of particles, for a certain grate configuration (frequency of reciprocation bar motion, length of travel, etc.). This information would contribute to the characterization and quantification of the mixing process. In the first test of the Markov chain model presented in this paper, we simulated only the bottom layer of the bed that is in contact with the bars of a reverse acting Martin grate. A transition matrix represented each section of the grate as a function of the number of successive reverse motions of the reciprocating bars of the grate. The pathway of a particular solid waste particle over a time series was then projected. It is hoped that this model will become a useful tool for designing future moving grate systems and increasing the combustion efficiency of existing ones.

Further studies of mixing modeling are planned in order to accurately predict the behavior of solid waste particles on the moving grate during the combustion process. These studies will include a model that has been developed to simulate a continuous variation of solid flow, that is a time series for the feeding of MSW in a chamber [10]. Also, experimental work is necessary to define the relationships between the probabilities of reverse and forward motion (i.e., \( p_r, r_r, p_f, r_f \)) and several parameters such as \( (R_r, S, R_, L, \alpha, H, \theta) \). The mixing behavior of MSW will be analyzed for other grate types. Some typical distributions such as Gaussian distribution, F-distribution and gamma distribution will be used in order to define particle sizes for each major component of MSW. The proposed one-layer Markov chain Model will be extended to cover the entire depth of the bed. It is expected that in the future this stochastic model will be combined with existing numerical bed and CFD models into a full mathematical model of the combustion chamber of a WTE facility.
Nomenclature

\( R_r \) feed rate of MSW in the inlet of the mass-burn WTE chamber
\( S \) particle size of MSW components
\( \rho \) particle density of MSW components
\( R_r \) frequency of reciprocating bars
\( L \) the length of the reciprocating bars travel
\( H \) height of the reciprocating bars
\( \theta \) angle of the reciprocating bars
\( \alpha \) angle of chamber bed decline
\( S(0) \) initial state vector (initial distribution of MSW)
\( n \) number of transition of the Markov chain (number of the stroke of reciprocating bars)
\( S(n) \) state vector after \( n \) transitions of the Markov chain (distribution of MSW after nth stroke of reciprocating bars)
\( P \) transition matrix
\( k \) cell number (positive integer: \( 1 < k < 16 \))
\( p_i \) probability of remaining in state \( i \) after one transition near the inlet
\( p_r \) probability of remaining in a state after one transition on the reciprocating bar
\( r_r \) probability of transiting to the adjacent state on the reciprocating bar
\( r_f \) probability of remaining in a state after one transition on the fixed bar
\( r_o \) probability of transiting to the adjacent state on the fixed bar
\( p_o \) probability of remaining in state 16 after one transition near the outlet
\( f_i \) function of \( R_r, S, \rho, R_r, L, \alpha, H \) and \( \theta \) to determine \( p_r \)
\( f_o \) function of \( R_r, S, \rho, R_r, L, \alpha, H \) and \( \theta \) to determine \( r_r \)
\( g \) function of \( R_r, S, \rho, R_r, L, \alpha, H \) and \( \theta \) to determine \( r_r \)

References


