Chaotic dynamics of the flood series in the Huaihe River Basin for the last 500 years

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Abstract

The Huaihe River Basin is one of the most flood-prone basins in China because it is frequently affected by collapses of the south levee of the Huanghe (Yellow River) over a long period in addition to its transitional climate and poor drainage topography. The flood series in the Huaihe River Basin for the last 500-year period is reconstructed using the ‘Atlas’ of historical recordings and annual hydrological data of the Basin, as well as the rational regional flood indices. The power spectrum structure of the flood series is similar to that of typical chaotic series and the attractor dimension (4.66) is larger than 2 and is a noninteger. Furthermore, chaotic dynamics of the flood series in the Huaihe River Basin with three dimensions and 2nd power is reconstructed according to chaos theory and the inverted theorem of differential equations. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Floods are natural disasters which occur frequently and result in deaths and damage to property. It is estimated that flood disasters are worldwide one of the most serious among 15 kinds of natural disasters which have a severe impact on human beings in the world. Indeed, floods constitute 27% of 594 recorded severe natural disasters in the world in 1996, while the death tolls are 56% of the total number of deaths and the economic losses are 57% of the total economic losses caused by flooding in 1996 (Jiang et al., 1997). Moreover, economic losses caused by floods have been increasing rapidly with the economic development, mounting population, accumulated properties and irrational land use in the flood plains of great rivers (Changnon, 1985).

China, situated in the eastern part of Eurasia—the biggest continent in the world, on the west coast of the Pacific Ocean—the largest ocean on the globe, with the Qinghai–Tibet Plateau—the highest and second largest plateau in the Earth, is strongly affected by monsoonal climate and terrain topography. Consequently, floods in China are frequent and serious. About 50% of the population lives and 70% of properties are located in the flood-affected zones. Besides, the annual economic losses caused by floods in China have also an obvious increasing trend (Li and Wang, 1991).

Furthermore, the Huaihe River Basin, located in the eastern part of China, is one of the most flood-prone basins in China because of its transitional climate from sub-tropical zone to warm temperate zone and...
from wet zone to semi-arid and arid zone, in addition to its poor drainage topography. In fact, floods have been and are still one of the most important restricting factors for the social stability and economic development in the Huaihe River Basin. For example, the great flood of the Huaihe River Basin and Taihe Basin in 1991 caused economic losses of 50 billion Chinese dollars (yuan) and nonharvest of more than 40 million acres of crops; these areas inhabited by more than 200 million people were declared disaster areas (CCIDNDR, 1991).

Two approaches, deterministic and stochastic, are generally applied in flood or water problem studies, while the combinations of the two approaches are often used. For example, it usually adds a stochastic item into a deterministic model in order to display the complexity of the flood system. Nevertheless, it is difficult, even impossible for the deterministic approach as well as the stochastic approach or the combination of both to reflect theoretically the complexity of the flood system which is nonlinear intrinsically. However, the discovery and continuing research of fractals and deterministic chaos gives flood research a new theorem, methods and opportunities (Porporato and Ridolfi, 1997).

Based on the annual flood grades in the Huaihe River Basin for the last 500-year period (1470–1991) according to the ‘Atlas of the Flood/Dryness in China for the Last 500-year Period’ (Atlas, MICMB, 1981) as well as numerous historical flood recordings and annual hydrological data, we reconstruct the flood series in the Huaihe River Basin according to rational regional flood indices. Moreover, after analyzing the chaos qualitatively and quantitatively and estimating the attractor dimension of the flood series in the Huaihe River Basin, we reconstruct a chaotic dynamics of the flood series in the Huaihe River Basin according to chaos theory and the inverted theorem of differential equations.

2. Regional setting

The Huaihe River Basin is situated in the eastern part of China and spreads out from the Tongbei-Dabi mountains in the west to the Yellow Sea in the east,
from the south levee of the Yellow River in the north to the Yangtze Basin in the south. The region of this study lies between 30°55′–36°30′N and 111°55′–122°5′E, with an area of 329,000 km² (Fig. 1).

The landscape of the Huaihe River Basin is characterized by a near level plain, a part of the North China Plain, with many lakes and depressions which have a great effect in lowering peak discharge and reducing flood hazards; except in western, southwestern and north-eastern areas with their mountains and rolling hills. Soils developed on the mountains and hills are brown earth, loam and sandy clay loam which are typically poorly irrigated; soils developed on the north of the Huaihe (the North Huaihe Plain) are mainly flooded silt soil and saline-alkali soil which are usually poorly drained, whereas soils developed on the south of the Huaihe are paddy rice which is well irrigated.

The climate of the Huaihe River Basin is chiefly characterized by monsoonal weather conditions with significant seasonal changes of precipitation which is mainly concentrated in the summer season with 60–70% of the annual precipitation; the conspicuous temperature variation of the basin, with high summer temperature 30 ⁰C (July) and low winter temperature 0 ⁰C (January), is obviously a result of the climatic continentality. Across the basin there is a gradual gradient in average annual precipitation from about 1000 mm in the southeast to less than 600 mm in the northwest, while the highest precipitation occurs in the inner mountain areas, is a result of the topography. The Huaihe River, with the west-eastward trending Qinling mountains in the west of the river, is an important physical geographic line which separates the subtropical zone south of the river and the warm-temperate zone north of the river, and accordingly the significantly different soil types, vegetation distribution, land use and other geographical characteristics in the north and south. For example, there is mainly paddy rice in the south of the river, whereas in the north there are dry crops like winter wheat and cotton.

Owing to the frequent collapse of the south levee of China’s second largest river—the Huanghe (Yellow River) since AD 1194, the original water systems of the Huaihe have been badly deranged and the drainage has been greatly impeded. As a consequence ‘small rains result in small hazards, heavy rains result in great floods, while no rains result in drought’. There had been 350 great floods in the basin during the period 1400–1900 and more than 70 average floods in each century according to historical records. For example, the great flood of 1921 in the Huaihe River Basin caused disasters over 36,500 km² with estimated damages exceeding 215 million traditional Chinese dollars (silver-yuan), while 24,900 lives were lost. The flood of 1931 in the basin was even more severe (Hu and Lu, 1993).

3. Reconstruction of the flood series in the Huaihe River Basin

3.1. Data

The data used in this paper is taken firstly from the Atlas. It has been systematically inspected and collated from numerous historical recordings obtained by the Chinese Meteorological Bureau in cooperation with about 30 major institutes and universities, and printed by the Science Press. The 5 flood/dryness grades are defined in the Atlas. They are flood, wetness, normal, dryness and aridity. Each grade has a uniform grading criterion. The flood/dryness grade for precipitation data are listed in Table 1.

The orthogonal function analysis shows that the data, presented in the Atlas on the flood/dryness grade and their distribution, can rationally reflect the basic circumstances of the precipitation of the summer season which is characterized by monsoonal climate. It can further reflect the flood/dryness situation in China.
for the last 500-year period from 1470 to 1979. This is of significant value for studying flood/dryness in China, especially the regional flood/dryness situation and their changes (Wang and Zhao, 1979).

Besides this source, numerous historical recordings of the Huaihe River Basin have been enriched and replenished for the same period and the annual flood grades are estimated according to the criteria of the Atlas. Meanwhile, the hydrological observations of the Huaihe River Basin from hydrological annals for each station are also replenished and further extended to 1991, while the flood grades for each station are obtained according to the indices of the Atlas.

3.2. Flood index in this study

Many researches on regional flood characteristics in China have been made according to the flood/dryness grade index of the Atlas (Zhang, 1981; Xu and Wang, 1981). However, two important factors, the flood degree and the flood area, were not sufficiently considered in these studies. Here we introduce the following regional flood index for considering the two important factors.

\[ I_i = \sum_{j=1}^{n} \frac{A_{ij}}{A} \times D_j \]  

where \( I_i \) is the flood index in the Hua River Basin in the \( i \)th year; \( A_{ij} \) is the flooding area of the \( j \)th grade in the \( i \)th year. \( A \) is the total area of the Hua River Basin; \( D_j \) is the weight of the \( j \)th flood grade in direct proportion to the flood degree.

According to the fact that there are two flood grades in the Atlas, the aforementioned regional flood index can be further simplified the following flood index.

\[ I_i = \frac{A_{i1}}{A} \times D_1 + \frac{A_{i2}}{A} \times D_2 \]  

(2)

where \( I_i \) and \( A \) are the same as in Eq. (1). \( D_1 \) and \( D_2 \) are the weights of the 1st and the 2nd flood grade, i.e. flood and wetness. Conspicuously, the flood grade should be higher if the flood degree is greater. Therefore, \( D_1 \) and \( D_2 \) are defined as 5 and 4, respectively, according to the 5 flood/dryness grades in the Atlas in this paper.

3.3. Flood series in the Hua River Basin

The flood series of the Hua River Basin for the last 500-year period is reconstructed based on the earlier regional flood index. It is shown in Fig. 2. Research on the series shows that the reconstructed flood series are rational and can well reflect the practical flood circumstances of the Basin (Zhou et al., 1996).

4. Analysis of the flood series

4.1. Power spectrum

Power spectrum analysis is often used in time series analysis for investigating regularity or periodicity of a discrete time series. Research shows that different time series have different power spectral characteristics (Yan, 1995; Tsonis, 1992). The power spectrum of a periodic function is discontinuous, it is called a linear spectrum and usually includes the fundamental frequency \( f_0 = (T)^{-1} \) and its harmonic waves
2/T, 3/T, ..., or its frequency divisions f_0/2, f_0/3, ..., the power spectrum of a quasi-periodic function has all frequencies whose ratio is irrational although it is also discontinuous and somewhat like that of a periodic function. Besides the basic frequencies f_0, f_1, ..., f_i, it includes kinds of assorted frequencies a_i f_j + b_j f_j, where a_i and b_j are any integers. However, the power spectrum of a chaotic series is continuous which is very different from that of periodic or quasi-periodic functions. This is helpful in analyzing qualitatively and judging whether the time series of a system is chaotic or not.

For convenience of comparing and analyzing, this paper puts the power spectrum of the flood series in the Huaihe River Basin and that of a typical periodic function like \( \sin \alpha \cos \beta \) and of the logistic map \( x_{n+1} = \mu x_n(1 - x_n) \) which is a kind of typical chaotic system with \( \mu = 3.99 \) and \( x_0 = 0.2 \) (Tsonis, 1992) into the same figure (Fig. 3). In Fig. 3, the length of each series is 512 \( (2^9) \). The discontinuity of the power spectrum of typical periodic function is obvious (Fig. 3a), while the continuity of the power spectrum of the logistic map is also distinct (Fig. 3b). However, because the power spectrum of the flood series in the Huaihe River Basin (Fig. 3c) is much similar to that of the logistic map, there is reason to qualitatively presume that the flood series in the Huaihe River Basin is chaotic.

It is difficult to distinguish from the power spectrum of a longer period to that of a chaotic series even though the series indicate chaos qualitatively from power spectra analysis. In addition, the power spectrum of a chaotic series and that of a stochastic series is also somewhat similar (Argyris et al., 1994). Therefore, further quantitative analysis of the flood series of the Huaihe River Basin is necessary. The chaosity of a time series can be analyzed by means of attractor dimension or other important parameters, such as the first Lyapunov exponent or the Kolmogorov entropy according to the chaos theory. These parameters are not only important for analyzing the chaosity of a time series, but have also an important role in revealing the characteristics of a chaotic dynamical system and are the basis for the prediction of chaotic series and the reconstruction of a chaotic dynamical system. They can be analyzed by phase space reconstruction in ergodic theory with the development of the fractal and chaotic theory (Tsonis, 1992; Rodriguez-Iturbe and Power, 1989; Eckmann and Ruelle, 1985).

4.2. Phase space reconstruction

It is generally believed that a time series of a variable is the output of a corresponding system which is so complex that the systematic information from a series is usually very limited and much useful information about systematic evolution may be lost or cannot effectively be extracted. Ergodic theory, which considers that one-dimensional time series contain much useful information about systematic evolution and traces of all variables that play a role in systematic evolution, abundantly enriches the traditional viewpoint. In fact, it is possible to grasp the systematic information from time series if one uses a rational method. Currently methods of extracting information of systematic evolution are being
enriched by the development of chaotic theory and its applied research. Two examples are the singular system approach (Broomhead and King, 1986) and the method of delays (Takens, 1981). The latter, which is often used is adopted in this study.

Assuming an one-dimensional time series, \( \{X(t_i), i = 1, 2, ..., \tilde{N}\} \), that is

\[
X(t_i) = x(t_0 + i\Delta t)
\]

where \( t_0 \) is the initial time point; \( \Delta t \) is the time interval; \( \tilde{N} \) is the length of the time series or the number of the observations, i.e. the sample capacity.

It is believed in ergodic theory that the previous time series of the state variables of a system contain traces of all variables that take part in systematic evolution. Therefore, a higher dimensional phase space \( R^m \) by introducing a time delay parameter \( \tau \) can be reconstructed and help to recover the dynamics of the original system (Takens, 1981). That is

\[
X_m(t_i) = \{x(t_i), x(t_i + \tau), ..., x(t_i + (m - 1)\tau)\}
\]

where \( i = 1, 2, ..., \tilde{N} - (m - 1)\tau; \tau \) is the time delay; \( m \) is the embedding phase space dimension. If the embedding phase space dimension \( m \) is big enough, the chaotic attractor of the system can be depicted.

4.3. Attractor dimension

Systems characterized by a chaotic dynamics move in a strange attractor with a fractal dimension \( D \) and at least one positive Lyapunov exponent (Froehling et al., 1981; Tsonis, 1992). The attractor dimension \( D \) can well be estimated by correlation dimension \( d \) and the reconstructed system is topologically equal to the original system so long as the dimension of the strange attractor is not very large (Takens, 1981; Tsonis, 1992). Furthermore, the correlation dimension \( d \) can be measured by the \( G-P \) method suggested by Grassberger and Procaccia (1983a,b). The main steps of the \( G-P \) method are:

1. For a time series \( \{X(t_i), i = 1, 2, ..., \tilde{N}\} \), selecting a rational time delay \( \tau \) and embedding dimension \( m \), and reconstructing a phase space \( R^m \), i.e. Eq. (4).

2. The correlation integral \( C(\varepsilon) \):

\[
C(\varepsilon) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} H(\varepsilon - ||Y_i - Y_j||)
\]

where \( \tilde{N} = \tilde{N} - (m - 1)\tau, N \) is the length of the phase trajectory; \( H(\theta) \) is the Heaviside step function; \( H(\theta) \) is equal to 1 if \( \theta \) is larger than or equal to zero, otherwise \( H(\theta) \) is equal to 0; \( || \) indicates a norm of the vector; \( ||Y(t_i) - Y(t_j)|| \) is the Euclidean distance between the phase coordinates \( Y(t_i) \) and \( Y(t_j) \) in \( d \)-dimensional phase space; \( \varepsilon \) is the distance threshold. It gradually becomes large while \( \varepsilon_0 \) should be very small.

3. The construction of the statistical relationship between \( C(\varepsilon) \) and \( \varepsilon \) is as follows. If the phenomenon is chaotic and the attractor is correctly reconstructed, \( C(\varepsilon) \) and \( \varepsilon \) are satisfied with the following power law when \( \varepsilon \) is small enough and \( \tilde{N} \) is big enough for a fractal system.

\[
C(\varepsilon) \propto \varepsilon^d
\]

where \( d \) is the correlation dimension.

4. Gradually increasing the phase space dimension \( m \) and repeating the first three steps, calculating the estimation of systematic dimension \( d(m) \) until the \( d(m) \) is not obviously changed while increasing the phase space dimension \( m \). That means the \( d(m) \) has tended to the saturation. The \( d(m) \) is the measurement of the correlation dimension and is further signed as \( d \) (Fig. 4).

It cannot be infinitely extended for a time series and
therefore determining the least length of a series for measuring \( d \) is important. It is estimated that a rational correlation dimension \( d \) can be obtained when the length of a series is about 500 or more for a system in which the attractor dimension is not very high (Judd, 1995; Yan, 1995). It has been suggested that the length of a time series should be longer (Ghilardi and Rosso, 1990). The embedding dimension \( m \) and the distance threshold \( \varepsilon \) are different for different series. The time delay \( \tau \) can be obtained by which the value of the autocorrelation function first reaches an eigenvalue \((0, 0.1, 1/e, \ldots)\) (Tsonis and Elsner, 1988; Tsonis, 1992). In this paper \( \lambda(\tau) = 0 \) is used.

According to this method, the correlation integral \( C(\varepsilon) \) is calculated with different \( \varepsilon \). Here, \( \varepsilon_0 = 0.2, \Delta\varepsilon = 0.2, \varepsilon_{\text{max}} = 20 \) for a certain embedding dimension \( m \) where \( m = 3, 4, \ldots \). When the embedding dimension \( m \) is 8, the correlation dimension \( d \) is not changed by increasing the embedding dimension \( m \). At that time the time delay \( \tau \) is \( 8\Delta t \). Here \( \Delta t \) is the unit time, which is 1 year. This means the saturated embedding dimension of the flood series in the Huaihe River Basin is 8. The correlation dimension is 4.66 (Fig. 4) (Zhou et al., 1998). The fact that the correlation dimension is larger than two and is a noninteger (fractal) quantitatively proves the chaos of the flood series in the Huaihe River Basin and is further testified by a positive Lyapunov exponent (Zhou et al., 1999).

5. Dynamical system reconstruction

The power spectrum and attractor dimension qualitatively and quantitatively analyze the chaos of the flood series in the Huaihe River Basin in which the strange attractor dimension is 4.66. It is necessary to reconstruct the nonlinear dynamical system if one wants to study further the chaos or nonlinear features of the system and thus obtain more details about the system.

There are some mathematical-physical models that are described by ordinary different equations in hydrology and the geosciences, such as the Saint-Venant equations and the Lorenz model. These models, based on ordinary differential equations, can be solved according to the equations and their initial conditions by using computing techniques. However, we only know a series of special solutions of a system in the case of a dynamical system which is described by a series of observations. This pertains to the inverted problem of differential equations. The dynamical system can be reconstructed from a series of observations by solving the inverted problem of differential equations according to the inverted theorem of differential equations (Huang and Yi, 1991; Peng et al., 1993).

5.1. Basic method

Let us assume the dynamical system is

\[
\frac{dX_i}{dt} = f_i(X_1, X_2, \ldots, X_n) \tag{7}
\]

where \( i = 1, 2, \ldots, m \), \( m \) is the number of the ordinary differential equations or the state parameters. It can be estimated by the chaotic attractor dimension, while the chaotic attractor dimension can be well approximated by the correlation dimension \( (d) \). The function \( f_i \) is a general nonlinear function of the state parameters \( X_1, X_2, \ldots, X_n \); \( n \) is the number of the state parameters and their combination on the right side of the differential equations.

For a time series of observations, we do not know how to describe the function \( f_i(X_1, X_2, \ldots, X_n) \) exactly. However, we do know a series of solutions of Eq. (7), i.e. \( X(j), j = 1, 2, \ldots, N \), where \( N \) is the number of parameters which can be figured out by the length of the series of observations \( N: \hat{N} = \hat{N} - (m - 1) \). Therefore we can rewrite Eq. (7) into a form of discrete difference equation.

\[
\frac{X_i(t+1) - X_i(t-1)}{2\Delta t} = f_i(X_1, X_2, \ldots, X_n) \tag{8}
\]

where \( t = 1, 2, \ldots, (N - 1) \).

For convenience, here we symbolize the parameters of the function \( f_i(X_1, X_2, \ldots, X_n) \) as \( Q_t \), where \( t = 0, 1, 2, \ldots, n \). \( Q_t \) can then be denoted by vector \( \mathbf{Q} \)

\[
\mathbf{Q} = (Q_0, Q_1, Q_2, \ldots, Q_n) = (1, X_1, X_2, \ldots, X_n) \tag{9}
\]

the corresponding coefficient is vector \( P_t \), where \( P_t = (P_0, P_1, \ldots, P_n) \). Therefore, the discrete difference equation (Eq. (8)) can be further transformed

\[
f_i(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{S} Q_t P_t \tag{10}
\]

where \( S = n + 1 \). Assuming that the observations can
comprise $M = N - 2$ equations, then Eq. (10) can then be rewritten by matrices or vectors

$$\mathbf{D} = \mathbf{Q}\mathbf{P}$$

while

$$\mathbf{D} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_M \end{bmatrix} = \begin{bmatrix} \frac{X_i(3) - X_i(1)}{2\Delta t} \\ \frac{X_i(4) - X_i(2)}{2\Delta t} \\ \vdots \\ \frac{X_i(N) - X_i(N - 2)}{2\Delta t} \end{bmatrix}$$

$$\mathbf{Q} = [Q_0, Q_1, \Lambda, Q_n] = \begin{bmatrix} Q_{10} & Q_{11} & \Lambda & Q_{1n} \\ Q_{20} & Q_{21} & \Lambda & Q_{2n} \\ M & M & \Lambda & M \\ Q_{M0} & Q_{M1} & \Lambda & Q_{Mn} \end{bmatrix}$$

If the matrix $\mathbf{Q}^\top\mathbf{Q}$ is invertible, we have

$$\mathbf{P} = (\mathbf{Q}^\top\mathbf{Q})^{-1}\mathbf{Q}^\top\mathbf{D}$$

where the symbol $(\cdot)^{-1}$ represents the inverse operation of a matrix.

5.2. Dynamical system reconstruction of the flood series

A dynamics that has at least five dimensions should be reconstructed according to the attractor dimension (4.66) of the flood series in the Huaihe River Basin. Also it should take account of the minimum integral dimension of the attractor. As a preliminary methodological research, here we assume the dimension $m = 3$, i.e. the state parameters are $X_1, X_2, X_3$, respectively. Moreover, let us again assume that the system which will be reconstructed only includes linear and second power nonlinear items. According to these assumptions, Eq. (7) can thus be transformed as follows:

$$\frac{dX_1}{dt} = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_1^2 + a_5X_2^2 + a_6X_3^2 + a_7X_1X_2 + a_8X_1X_3 + a_9X_2X_3$$

$$\frac{dX_2}{dt} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_1^2 + b_5X_2^2 + b_6X_3^2 + b_7X_1X_2 + b_8X_1X_3 + b_9X_2X_3$$

$$\frac{dX_3}{dt} = c_0 + c_1X_1 + c_2X_2 + c_3X_3 + c_4X_1^2 + c_5X_2^2 + c_6X_3^2 + c_7X_1X_2 + c_8X_1X_3 + c_9X_2X_3$$

In Eq. (17), $a_i, b_i, c_i, i = 1, 2, \ldots$, are the coefficients of the state parameters which are unknown as yet. However, the state parameters $X_1, X_2$ and $X_3$ and their ‘velocities’ $dX_1/dt, dX_2/dt$, and $dX_3/dt$ can be estimated from the flood series in the Huaihe River Basin according to the phase space reconstruction theorem (Takens, 1981). Based on the previous assumptions, we write the original series in three dimensions.

$$\begin{cases}
X_1 : x(t_1) & x(t_2) & x(t_3) & \ldots & x(t_N - 2\tau) \\
X_2 : x(t_1 + \tau) & x(t_2 + \tau) & x(t_3 + \tau) & \ldots & x(t_N - \tau) \\
X_3 : x(t_1 + 2\tau) & x(t_2 + 2\tau) & x(t_3 + 2\tau) & \ldots & x(t_N) 
\end{cases}$$

where $\tau$ is the time delay, here $\tau = 1$; $k$ is the length of evolving step where $k = 1, 2, \ldots$. Let us consider the first line in Eq. (18) as a series of
values of the first state parameter $X_1$, the second is $X_2$ and the third $X_3$, respectively. The dynamical system (Eq. (8)) can be transformed into the following discrete difference equations (Eq. (19), where $i = 1; \Delta t = 1$) after putting those state parameters into Eq. (17).

$$
X_i(t + 1) - X_i(t - 1) = a_0 + a_1X_1(n) + a_2X_2(n) + a_3X_3(n) + a_4X_1^2(n) + a_5X_2^2(n) + a_6X_3^2(n) + a_7X_1(n)X_2(n) + a_8X_1(n)X_3(n) + a_9X_2(n)X_3(n)
$$

(19)

Corresponding to Eq. (12), we have the vector $D$. When the items on the left side of Eq. (19) become known, and the items on the right side can also be solved according to the embedded three-dimensional time series. We denote vector $Q$

$$
Q = (Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, Q_9)
$$

$$
= \{1, X_1(n), X_2(n), X_3(n), X_1^2(n), X_2^2(n), X_3^2(n), X_1(n)X_2(n), X_1(n)X_3(n), X_2(n)X_3(n)\}
$$

(20)

If we combine Eqs. (19) and (20) with Eq. (13), the vector $Q$ (Eq. (13)) can be obtained, where $n = 9, M = N - 2$.

Returning to Eq. (11) and the vector $P$, the coefficients on the right side of the equation (Eq. (11)) are still unknown. They can be denoted by matrix $A$.

$$
A = [a_0, a_1, \ldots, a_n]
$$

(21)

Solving Eq. (16), we can estimate the coefficients of matrix $A$.

Similarly, the coefficients of matrices $B = (b_1, b_2, \ldots, b_n)^T$ and $C = (c_1, c_2, \ldots, c_n)^T$ of the state parameters $X_2$ and $X_3$ can also be estimated, respectively.

5.3. The reconstructed dynamical system

The set of the coefficients in the dynamical system of the flood series in the Huaihe River Basin are estimated according to the aforementioned method. The results are shown in Table 2. From Table 2, we can see that some coefficients are small while others are relatively large. It is possible that the large coefficients may not have large contribution to the systematic evolution. Therefore, we further estimate the relative variance of the corresponding coefficients in Table 2 for selecting coefficients rationally. The relative variance of the coefficients is estimated by

$$
R_i = \frac{1}{N} \sum_{j=1}^{N} \left[ \frac{T_i^2}{\sum_{i=0}^{n} T_i^2} \right]
$$

(22)

where $R_i$ is the relative variance of the $i$th coefficient;
\( N \) is the length of the flood series, \( N = 522; \) \( T_i \) is the value of the \( i \)th item on the right side of Eq. (19); \( n = 9 \).

From Table 2, we can see that the varied range of the relative variances is very large. Owing to the nonlinearity of the flood system and the importance of the significant flood events, we consider the items that the relative variance is large should be selected for making the reconstructed dynamical system embody significant flood events. Therefore, the items that the relative variances are larger than \( 0.1 \times 10^{-1} \) are selected according to Table 2. The reconstructed dynamical system of the flood series in the Huaihe River Basin is

\[
\begin{align*}
\frac{dX_1}{dt} &= a_0 + a_1 X_1 + a_4 X_1^2 + a_5 X_2 + a_7 X_1 X_2 + a_8 X_1 X_3 \\
\frac{dX_2}{dt} &= b_0 + b_2 X_2 + b_3 X_2^2 + b_7 X_1 X_2 \\
\frac{dX_3}{dt} &= c_0 + c_3 X_3 + c_6 X_2^2 + c_8 X_1 X_3
\end{align*}
\]

(23)

where \( a_0 = 2.7153, \ a_1 = -0.8239, \ a_4 = 0.2014, \ a_5 = 1.0765, \ a_7 = -1.0024, \ a_8 = -0.4642; \ b_0 = 1.7032, \ b_2 = -3.6701, \ b_3 = 0.4603, \ b_7 = -2.5420; \\
\ c_0 = 2.7635, \ c_3 = -1.4763, \ c_6 = 2.0338, \ c_8 = -1.2074. \)

We can find from Eq. (23) that the reconstructed dynamics of the flood series in the Huaihe River Basin has not only the constants and linear items, but has also several nonlinear items. It is even more complex than the famous Lorenz model or other typical chaotic systems, like the Rössler system.

6. Discussion and conclusions

The study of nonlinear phenomena has taken great progress in the last century, especially with the development of the fractals and chaotic theory which is one of the important scientific concepts in the 20th century and their applications in the last two decades. Fractals have a special ability to quantify geographic or geometric phenomena by means of computer techniques and are applied in many fields, including water resources research and the geosciences. Chaos theory further deals with the problems inherent in a system, such as its nonlinear characteristics, evolution, dynamical behavior and prediction. Chaotic studies are necessary for a complex or nonlinear system considering that almost all real geo-systems are nonlinear and complex. In fact, with the development of the chaotic theory and in the light of these ideas and methods, chaotic study in geosciences has not only been restricted to the basic but important contents, like the chaotic judgment of a system and the estimation of the chaotic attractor dimension, but it has also come into the practical study stage, such as the characteristics of chaotic evolution, dynamical system reconstruction (DSR) and nonlinear prediction (Wang and Zhao, 1993; Porporato and Ridolfi, 1997). The work presented here is based on these concepts.

In this study, we have reconstructed the flood series in the Huaihe River Basin for the last 500-year period on the basis of the Atlas and numerous historical recordings, using the annual data of the basin and the rational regional flood index. According to the reconstructed series, the chaoticity of the flood series in the Huaihe River Basin is qualitatively and quantitatively analyzed by means of the power spectrum structure of the flood series which is similar to that of a typical chaotic series with a noninteger attractor dimension larger than two. Furthermore, a preliminary study of the chaotic dynamics of the flood series in the Huaihe River Basin which is three-dimensional and 2nd power nonlinear is reconstructed according to chaos theory and the inverted theorem of differential equations. Nevertheless, the detailed chaotic behavior of the dynamics of the flood series in the Huaihe River Basin should be further studied using a higher dimensional and power dynamics according to the attractor dimension of 4.66 of the flood series in the Huaihe River Basin and a higher power, such as the third, fourth or higher power.

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