Time Domain VALEN Calculations

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• Calculations performed with **VALEN** computer code **in the time domain**.

• **Resonant Field Amplification**

• **Time evolved calculations**, basic passive performance and results with idealized feedback

• **Time evolved calculation**, adding realistic effects to feedback circuits: **noise and time delay**.
VALEN Formulation

VALEN uses a thin shell lumped circuit finite element approximation to model induced currents in passive structures. The resulting equations are matrix L & R 'mesh' circuit equations, conducting thin shells, arbitrary thin coils, and magnetic sensors may be included in this formulation. Sensors are modeled via a mutual inductance to the problem variables, i.e.,

\[
\{\Phi_{sensor}\} = [M]\{I\}
\]

feedback rules may be defined among the sensors and coils.

\[
G_p \Phi_s(t - \tau_p) + G_d(-\dot{\Phi}_s)(t - \tau_d) = V_{coil}(t)
\]
A plasma mode can be included, see Boozer, Physics of Plasmas, 5, page 3350, (1998)

\[
\begin{align*}
\begin{bmatrix} L_{ww} \end{bmatrix} \{ I^w \} + \begin{bmatrix} M_{wp} \end{bmatrix} I^d + \begin{bmatrix} M_{wp} \end{bmatrix} I^p &= \{ \Phi^w \} \\
\begin{bmatrix} M_{pw} \end{bmatrix} \{ I^w \} + LI^d + LI^p &= \Phi \\
LI^p &= (1 + s)\Phi
\end{align*}
\]

The plasma mode is represented by a current distribution in a control surface surrounding the unperturbed plasma, the strength of this mode is represented by the parameter \( s \) is the normalized mode energy, when \( s<0 \) stable, when \( s>0 \) unstable, and marginal stability.

From a plasma equilibrium, we obtain \( \delta W \) and the mode’s B-normal on the plasma surface from \textbf{DCON} ( A. Glasser's code ).
Resonant Field Amplification

A. Boozer predicted Resistive Wall Modes would amplify magnetic field (e.g. errors) see Physics of Plasmas 10, pg 1458 (2003))

The VALEN formulation handles this via an extra field source

\[
\begin{align*}
[ L_{ww} ] \{ I^w \} + [ M_{wp} ] I^d + [ M_{wp} ] I^p + [ M_{wD} ] \{ I^D \} &= \{ \Phi^w \} \\
[ M_{pw} ] \{ I^w \} + L I^d + L I^p + [ M_{pD} ] \{ I^D \} &= \Phi \\
L I^p &= (1 + s)\Phi
\end{align*}
\]

\[
\begin{align*}
\{ \dot{\Phi}_w \} + [ R_{ww} ] \{ I^w \} &= \{ V \} \\
\{ \dot{\Phi} \} + [ R_d ] \{ I^d \} &= \{ 0 \}
\end{align*}
\]

(resonant field amplification) in the sensors is given by

\[
\begin{align*}
\left( [ M_{sw} ] - [ M_{sp} ] \frac{(1 + s)}{sL} [ M_{pw} ] \right) \{ I^w \} + \left( [ M_{sp} ] - [ M_{sp} ] \frac{(1 + s)}{sL} L \right) I^d + \\
\left( [ M_{sD} ] - [ M_{sp} ] \frac{(1 + s)}{sL} [ M_{pD} ] \right) \{ I^D \} = \{ \Phi^s \}
\end{align*}
\]
As ‘s’ approaches 0- we have two effects:
1) near singular behavior (amplification)
2) RWM mode response gets slower!

We expect similar behavior approaching marginal stability in a rotationally stabilized RWM above the no-wall limit.

We demonstrate RFA in a VALEN model of DIII-D
1) use ‘C-coil’ to generate a resonant field
2) use poloidal sensors with small direct coupling to the C-coil
3) examine sensor signal as ‘s’ varies
Weakly Damped Modes Slow to Approach Steady State Values

Sensor Signals

\[ \beta_n = 1.42 \]
\[ s = -0.1 \]

Sensor signals

\[ \beta_n = 2.85 \]
\[ s = -0.01 \]

\[ -\left[ M_{sp} \right] \left( 1 + s \right) \left[ M_{pD} \right] \]

Weakly Damped Modes Slow to Approach Steady State Values
Sensor Flux for Damping Rates Showing Resonant Field Amplification

<table>
<thead>
<tr>
<th>S</th>
<th>$\beta_N$</th>
<th>Damping Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>1.42</td>
<td>-0.012 sec</td>
</tr>
<tr>
<td>-0.01</td>
<td>2.85</td>
<td>-0.058 sec</td>
</tr>
<tr>
<td>-0.001</td>
<td>2.98</td>
<td>-0.546 sec</td>
</tr>
</tbody>
</table>

DIII-D Results

![Graph showing sensor flux and damping time results.](image)
VALEN Eigenvalue (growth rate) and transient calculations are consistent for zero time delay in the feedback logic.

Example from VALEN ITER analysis:

Growth rate predicted
By eigenvalue analysis
Do transients at s=0.1

Expected growth rates
Observed in transient calculations

Data from "ITER.12.2002"

Data from "ITER.cs5"

Time [s] 0.00 0.01 0.02 0.03 0.04

Growth rate [1/s]

flux in sensor 6 [v*s]

S=0.1

passive (no blankets)

passive with blanket

gain = 10^7

gain = 10^8 [v/v*s]

gain = 10^8 [v/v*s]

Gp=10^7 reduced growth rate

Gp=10^8 stabilized!
DIII-D I-Coil Feedback Performance

Conversion from $s$ to $\beta$

$$\beta_{\text{normal}} = \frac{\beta - \beta_{\text{no\_wall}}}{\beta_{\text{ideal\_wall}} - \beta_{\text{no\_wall}}}$$

<table>
<thead>
<tr>
<th>$s$</th>
<th>$\beta$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15849</td>
<td>4.49</td>
<td>97.0</td>
</tr>
<tr>
<td>0.15634</td>
<td>4.48</td>
<td>95.8</td>
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<td>0.15418</td>
<td>4.48</td>
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<td>0.14987</td>
<td>4.46</td>
<td>92.2</td>
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<td>0.14125</td>
<td>4.43</td>
<td>87.4</td>
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<td>0.13357</td>
<td>4.40</td>
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<td>0.12589</td>
<td>4.38</td>
<td>78.9</td>
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<tr>
<td>0.11295</td>
<td>4.33</td>
<td>71.7</td>
</tr>
<tr>
<td>0.10000</td>
<td>4.29</td>
<td>64.5</td>
</tr>
</tbody>
</table>
Simulated Noise on DIII-D RWM Poloidal Sensors

**Broadband noise** was modeled as Gaussian random number with standard deviation **1.5 G** about 0 mean and frequency **10kHz**.

To the broadband noise **ELMs** were added as additional Gaussian random numbers from **6 to 16 Gauss** approximately every 10 msec with +/- chosen with **50% probability**.
DIII-D I-Coil Feedback Current Simulation with Sensor Noise for Range of $\beta_N$

$L=60 \text{ mH and } R=30 \text{ mOhm with Proportional Gain } G_p=7.2\text{Volts/Gauss}$
Resonant Amplification of Noise Limits Feedback when Approaching Ideal Limit

Maximum control coil current and voltage as function of $\beta_{\text{normal}}$
Effects of Noise on Feedback Dynamics

Sensor Flux

L=60 mH and R=30 mOhm DIII-D I-Coil Feedback model with Proportional Gain $G_p=7.2$Volts/Gauss

turn on FB at $t = 1.65 \text{ ms}$
VALEN may model time delays in Feedback Performance
Examine performance of DIII-D I-coil system

VALEN predictions
Eigenvalue analysis
Zero delay time
Results vs. ‘s’

VALEN predictions
Eigenvalue analysis
Zero delay time
Results vs. normalized beta n

Data from "DIII-D.10.2003.newR&L"
BAISC problem, start feedback at \( t = 0.35 \) ms, beta-\( n = 4.76 \)
Feedback defined by:
Poloidal sensor flux \( \times \) gain = control coil voltage
Stabilized behavior!

No time delay! I.e.,

\[
G_p \Phi_p(t) = V_{cc}(t)
\]

Coil current & sensor flux
Sensor flux & coil voltage
Small time delay, start feedback at $t = 0.35$ ms, beta-$n = 4.76$
Feedback defined by:
Poloidal sensor flux(delayed) * gain = control coil voltage

$$G_p \Phi_p (t - 0.0001) = V_{cc} (t)$$

Delay = 0.10 ms
increase time delay, start feedback at $t = 0.35$ ms, $\beta_n = 4.76$

Feedback defined by:
Poloidal sensor flux(delayed) * gain = control coil voltage

$$G_p \Phi_p (t - 0.00015) = V_{cc}(t)$$

Delay = 0.15 ms

Start feedback in control coil
Start feedback in sensor

Data from "DIIID.10.2003.01.delaytime"
Stability depends on plasma growth rate and time delay in feedback!
Critical delay time as a function of plasma beta

\[ \tau_{\text{critical}} = 1.2224 \times 10^{-3} - 1.2198 \times 10^{-3} \Delta \beta_n \]

\[ C_\beta = \frac{\beta_n - \beta_n|_{\text{no-wall}}}{\beta_n|_{\text{ideal wall}} - \beta_n|_{\text{no-wall}}} \]
Summary

- RFA time dependent modeling in qualitative with DIII-D results.
- Time dependent feedback results consistent with eigenvalue analysis.
- Noise simulation show performance limitation when approaching feedback stabilized marginal stability limit.
- Time dependent feedback simulation predict critical time delay for RWM stabilization.