The feedback phase instability in the HBT-EP tokamak

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Abstract. Observations of a performance limiting feedback phase instability in the HBT-EP tokamak are reported. The phase instability consists of a rapid growth of the phase difference between an \( m/n = 2/1 \) tearing mode and an external resonant magnetic perturbation. Observations of mode angular dynamics during phase instability test discharges show good agreement with theoretical estimates of the phase instability timescale. The phase instability limits feedback performance in HBT-EP by decreasing the feedback loop’s phase accuracy as gain increases.

1. Introduction

The \( m/n = 2/1 \) tearing mode instability is a precursor to major disruptions in tokamaks. The HBT-EP tokamak [1] has been used to investigate active methods of stabilizing the \( 2/1 \) mode by applying resonant magnetic perturbations produced with external coils. Suppression of the \( 2/1 \) mode using resonant magnetic perturbations requires aligning the phase difference between the mode and the perturbation so that the perturbation opposes the \( 2/1 \) mode. A constant frequency perturbation will ultimately fail to suppress the mode, because the mode will change its rotation frequency until it aligns itself with the perturbation in a destabilizing phase [2]. Feedback can be used to suppress the \( 2/1 \) mode by dynamically controlling the perturbation phase to maintain a stabilizing feedback phase difference.

The phase instability can limit the performance of a tearing mode feedback system using external control windings if it causes the mode phase to change faster than the bandwidth of the feedback loop [3]. The phase instability was observed during feedback experiments in the DITE tokamak, but it did not seem to affect feedback performance [4]. However, the slowest phase instability timescale in HBT-EP is observed to be about ten times faster than the slowest timescale reported at DITE, and in HBT-EP a decrease in feedback phase accuracy is observed as loop gain is increased. This article extends our investigations of the phase instability by comparing experimental measurements with theoretical predictions and by quantifying the effect of the phase instability on feedback phase accuracy as gain is increased, with other settings held constant. The theoretical model that fits our data well also predicts timescales consistent with reported measurements at DITE.

The feedback experiments on rotating \( m/n = 2/1 \) islands in HBT-EP [5] use two sets of modular external \( m = 2 \) saddle coils arranged to approximate quadrature \( 2/1 \) windings. One set of saddle coils is driven with a cosine phase current and the other set is driven with a sine phase current, producing a rotating \( 2/1 \) perturbation. The saddle coils have small toroidal width (2 × 6° per phase), and their location over insulated quartz sections of the vacuum vessel and between toroidal gaps in the adjustable conducting shells of HBT-EP allows fast penetration of applied magnetic fields. Each phase is driven by a 10 MW linear power amplifier that provides up to 600 A at frequencies under 25 kHz. Synthesized control waveforms are produced by a digital signal processing (DSP) computer with a 100 kHz output sampling rate. This feedback apparatus has been used to demonstrate open loop control of \( 2/1 \) mode rotation with applied resonant torques [1, 6].

HBT-EP exhibits MHD oscillations attributed to a rotating \( 2/1 \) island (\( q_a \approx 2.5 \)), which grows and saturates for 2–5 ms prior to disruption. The saturated island rotates naturally with the electron diamagnetic drift at \( f \approx 10 \) kHz, but the island will lock to a rotating perturbation applied with the saddle coil set and follow changes in the perturbation frequency. When the perturbation frequency is close to the natural mode frequency, the mode locks to the perturbation with a driving or positive feedback phase difference (\( \Delta \Phi \approx 0^\circ \)).

In Section 2 we describe the method for producing and measuring the phase instability by locking the saturated \( 2/1 \) mode, and compare our measurements with theoretical estimates of the phase instability timescale. In Section 3 we measure the effect of the phase instability on feedback experiments at different levels of feedback loop gain.
2. Phase flip experiments

Creating a phase instability in an open loop configuration (described in Ref. [4]) involves locking the 2/1 mode to an applied perturbation, then generating a $\pm 180^\circ$ step in the perturbation phase. The mode is then in an unstable negative feedback phase ($\Delta \Phi \approx 180^\circ$) and is seen to rapidly change its phase to re-lock to the applied perturbation. The time it takes for the mode to return to the initial locked phase is proportional to the phase instability timescale.

Mode amplitude and phase are measured by a 15 coil poloidal Mirnov array located approximately 2 cm from the plasma surface. Quadrature (cosine and sine) $m = 2$ signals are computed with a least squares fit. The perturbation amplitude and phase are monitored by Rogowski coils attached to the saddle coil circuits.

The mode response to a perturbation phase flip is shown in Fig. 1. After the phase flip the 2/1 island re-locks to the perturbation in approximately one oscillation period. In this discharge the perturbation phase is flipped approximately $180^\circ$ and the island rotation slows and then re-locks. The mode amplitude after re-locking is slow to recover its pre-flip value. This is modelled well by adding a frequency dependent stabilizing term to the dynamic island width model [5]. Shifts of mode frequency away from its natural frequency cause transient attenuation of the mode amplitude.

The island dynamical model in Ref. [3] treats the island moment of inertia as the constant fraction of the plasma that is ‘free to rotate’. Using this model the theoretical phase instability timescale for the 2/1 mode is

$$\tau_{2/1} = R_0 r_s \sqrt{\frac{2\rho_0}{I_A B_\theta r_b}}$$

where $r_b$ is the radius of helical control current $I_A$ and $B_\theta$ is the poloidal field perturbation due to the 2/1 mode at the $q = 2$ rational surface radius $r_s$. $R_0$ is the major radius of the plasma and $\rho_0$ is the mass density of the rotating portion of the plasma. In another model [7] the island moment of inertia is a function of island width and the phase instability timescale is proportional to $B_\theta^{-1/4}$. The mode amplitude in these experiments did not span a large enough range to distinguish one model from the other. A range of $\tau_{2/1}$ was obtained primarily by changing $I_A$.

![Figure 1](image1.png)

Figure 1. Saddle coil perturbation currents, $m = 2$ mode signals and feedback phase difference from a phase flip experimental discharge (No. 12976). The perturbation phase flip ($180^\circ$) occurs just before 4.3 ms. The mode slows to re-lock with the regressed perturbation. The re-locking period is within the shaded area.

![Figure 2](image2.png)

Figure 2. Measured re-locking period versus theoretical estimate of phase instability timescale from measured plasma parameters.

The theoretical timescale for each phase flip discharge is calculated from Eq. (1) using plasma diagnostic measurements averaged over the initial locking period prior to the phase flip. Plasma density
Feedback phase instability in HBT-EP is measured by microwave interferometry. The helical control current is computed from the saddle coil current by numerically modelling the magnetic fields produced at the mode rational surface by the saddle coil set [8].

Measured re-locking periods are plotted against estimated timescales for several phase flip discharges in Fig. 2. The proportionality between the phase instability timescale and the re-locking period is dependent on the value of $\Delta \Phi$ at the time of the phase flip. Fluctuations in $\Delta \Phi$ during the initial locking period cause scatter in this proportionality.

3. Feedback accuracy experiments

The feedback experiments in HBT-EP use a phase rotation algorithm [9] implemented in the DSP computer to produce a rotating control perturbation from sine and cosine $m = 2$ input signals. The quadrature input is computed in real time from a set of four poloidal Mirnov coils [8]. The saddle coil currents that produce the rotating control perturbation are calculated using the following feedback algorithm:

$$
\begin{pmatrix}
I_{\cos}(n) \\
I_{\sin}(n)
\end{pmatrix} = G \begin{pmatrix}
\cos \delta & -\sin \delta \\
\sin \delta & \cos \delta
\end{pmatrix} \begin{pmatrix}
\tilde{B}_{\cos}(n-1) \\
\tilde{B}_{\sin}(n-1)
\end{pmatrix}.
$$

(2)

The algorithm amplifies the $m = 2$ mode signal by gain $G$ and rotates it by $\delta$ radians. Due to the latency of the DSP hardware, the output sample $n$ is delayed by 10 $\mu$s from the input sample $n - 1$.

The effect of the phase instability on feedback performance was investigated by producing closed loop feedback discharges at a constant $\delta$, but at different levels of gain. Feedback attenuation of the mode was small in these discharges so $\tilde{B}_0$ was nearly constant over each feedback period, therefore $\tau_{2/1}$ was inversely proportional to $\sqrt{G}$ (since $G \equiv \langle I_A/\tilde{B}_0 \rangle$). As feedback gain increases, the phase instability timescale approaches the latency time of the feedback loop, and we observe a measurable decrease in feedback phase accuracy.

Feedback phase accuracy is quantified by measuring the deviation of the mean achieved feedback phase difference $\Delta \Phi$ from the expected phase difference programmed by selecting $\delta$. The average mode frequency changes proportionally to $\sqrt{G}$ (in this case it increases), and this increases the loop phase error due to DSP latency. The variance of the feedback phase and the deviation of its mean are plotted in

![Figure 3. Variance of the feedback phase difference $\Delta \Phi$ and deviation of its mean from the programmed feedback phase difference $\delta$, versus $\sqrt{G}$. In this plot $G \equiv \langle I_A/\tilde{B}_0 \rangle$.](image)

Fig. 3. The variance of $\Delta \Phi$ increases with $\sqrt{G}$ and the deviation of the mean appears to increase with $G^2$. The scatter in the phase deviation plot is due to the shot to shot variance of $\Delta \Phi$.

4. Summary

Measurements of the re-locking period of the phase instability in HBT-EP show good agreement with theory. The phase instability has been experimentally observed as a performance limiting factor in tearing mode feedback experiments. The phase instability increases the rate of change in mode frequency as the amplitude of the feedback perturbation increases. This affects the feedback phase accuracy in two ways. It shifts the time averaged feedback phase away from its programmed value and it increases the level of instantaneous fluctuations of the feedback phase.

References